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REGIONAL FREQUENCY ANALYSIS FOR EXTREME RAINFALL IN SICILY

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1. Introduction

Reliability of hydraulic and hydro-geological risk evaluations in a fixed region mostly depends on the knowledge of intensity-duration-frequency relationship (IDF) of extreme rainfall events.

In fact, in order to evaluate peak flood events in ungaged sites and considering the short sample size of extreme flood events temporal series, it is often preferred to recur to pluviometric determinations and use indirect rainfall-runoff models.

From a statistical point of view, the actual necessity to evaluate a return period higher than the sample size showed the inadequacy of inferential statistical techniques. These techniques are inappropriate to determinate correctly the distribution right tail, leading to modern regionalization techniques.

The spatial-temporal stationarity hypothesis within statistically homogeneous wide areas, adopted by such techniques, allows the information transfer from space to time. The latter solution allows high sample size, ensuring the moments and the reliable probability distribution estimations. The many and diversified regionalization techniques proposed in the last years may nevertheless conduct to very different results, according to different modalities and approaches adopted in their implementation.

The "index flood"[1] method is the regional technique widely used in Italy thanks to the commitment of the Italian National Research Group for the Prevention of Hydro-Geological Disaster (GNDCI), belonging to CNR [5, 6, 7]. This group has developed a special operative programme for the definition of suitable methodologies and uniform procedures to estimate intense rainfall and peak flood in Italian country [5, 6, 7, 23], developing a national research project, called VAPI (VAlutazione delle Piene in Italia), based on the use of TCEV (Two-Component Extreme Value distribution) probabilistic model [1, 20]; in Sicily the mentioned study was delivered in 1993 by the GNDCI research unit, supervised by prof. Ignazio Melisenda Giambertoni.

In this manuscript we update the regional study carried out by the Sicilian research unit about short duration extreme rainfall, introducing also the sequent recent regionalization techniques:

- the probabilistic regional model based on L-moments, assuming that the latter statistics are constant in homogeneous regions [11, 12, 13];
- the parametric method MGs, introduced by Maione et al. [2, 16, 17], which considers the spatial variability of the conventional moments higher than the first order.

2. Regional models

2.1 *TCEV model*

This model can be classified as an "index flood" method, whose main assumption is that the hydrologic variable X, within a statistically homogeneous region, has the same frequency distribution F(x=X/I), apart from a scale factor I(X), called index flood. I(X) is usually the at-site mean of the probability distribution, though any location parameter of the distribution may be used.

In a homogeneous region, the variable X(T), for a fixed return period T, is estimated as the product between the index flood and the dimensionless quantile

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[1] The term "index flood" comes from early method applications to flood data in hydrology, but the procedure can be used with other kind of data, such as extreme rainfall.

x(T) of the regional frequency distribution F(x):

$$X(T)=I(X)\,x(T) \tag{1}$$

The function x(T) is called "growth curve".

The regional model based on TCEV (two-component extreme value) distribution is valid under the hypothesis of stationarity and spatial-temporal independence of the observations [23].

The TCEV distribution explains the population of annual maximum value as originating from two different populations, the first one caused by ordinary events and the second one caused by extreme events [5, 19, 23]. The TCEV expression is:

$$F(X) = \exp[-\lambda_1 \exp(-X/\vartheta_1)-\lambda_2 \exp(-X/\vartheta_2)] \tag{2}$$

where the parameters λ_1, λ_2 are the mean number of annual events respectively of the ordinary component and the outlying one (shape parameters, with $\lambda_1 >> \lambda_2$) and ϑ_1, ϑ_2 are scale parameters, respectively of the ordinary component events and the extraordinary one ($\vartheta_2 >> \vartheta_1$) [20].

The distribution probability F(x) of the variable x=X/I, is:

$$F(x) = \exp[-\lambda_1(\exp(\frac{\mu}{\theta_1}))^{-x} - \Lambda * \lambda_1^{1/\Theta*} \exp((\frac{\mu}{\Theta * \theta_1}))^{-x}] \tag{3}$$

where the shape parameter $\Theta^*=\theta_2/\theta_1$ and the scale parameter $\Lambda^*=\lambda_2/\lambda_1^{1\Theta^*}$ depend only on the distribution coefficient of skewness γ [5, 19, 23].

A hierarchical procedure of the regional parameters estimation, based on three successive levels, derives from the further observation that the coefficient of variation CV of the TCEV distribution depends on the parameters Λ^*, Θ^* e λ_1 [23].

The first level of regionalization implies the research of regions with γ constant, which involves Λ^* and Θ^* (estimated by the maximum likelihood method) constant, in the same region.

The second level of regionalization requires the research of sub-regions with CV constant, which involves λ_1 constant in addition to Λ^* and Θ^* constant.

The third level of regionalization consists in the definition of empirical relations able to estimate the index value.

2.2 *LM model*

Among the index flood methods recently introduced, we find the regional probabilistic model based on the use of the linear moments, called L-Moments (afterwards called LM).

Representing an evolution of the probability weighed moments introduced by Grenwood et al. [10], LM are estimated as linear function of the data respect to conventional moments, which are expressed by the elevation to power of the data. This implies that:

- LM are less sensitive than conventional moments to the presence of outliers in a sample [13, 24];
- LM estimators are unbiased for all sample sizes and all distributions, also in the case of highly skewed distributions, against conventional moments [24].

Besides LM, respect to conventional moments, allow a more robust estimation of frequency distribution parameters, especially for small samples [13, 24] and a more efficient statistical parameter estimation than the maximum likelihood method [13]. In particular the latter method, applied to small samples, loses in accuracy [3, 13].

Vogel [24] showed that the use of the LM is preferable in the case of highly skewed distributions if the probability distribution identification of a data sample is made by graphic comparison between the moments empiric values and the moments theoretical distribution.

The first four L-moments are [13]:

$$\begin{aligned}
\lambda_1 &= E[X_{1:1}] \\
\lambda_2 &= \frac{1}{2}E[X_{2:2} - X_{1:2}] \\
\lambda_3 &= \frac{1}{3}E[X_{3:3} - 2X_{2:3} + X_{1:3}] \\
\lambda_4 &= \frac{1}{4}E[X_{4:4} - 3X_{3:4} + 3X_{2:4} - X_{1:4}]
\end{aligned} \tag{4}$$

where E is the expected value and $X_{i,j}$ is the variable value, growing ordered, of the sub-sample with size j drawn by the sample considered.

Analogously to traditional moments, λ_1 is a location measure of the distribution and coincides with the sample mean, λ_2 is a scale measure of the distribution and is always greater than, or equal to, zero [13].

The L-moments ratio are dimensionless quantities and are defined as follows [13]:

$$\tau = \lambda_2/\lambda_1\text{, called L-CV} \tag{5'}$$

$$\tau_r = \lambda_r/\lambda_2\text{, where } r = 3, 4, \ldots \tag{5''}$$

where τ (L-CV), τ_3 (L-skewness) e τ_4 (L-kurtosis) are respectively measures of variation, skewness and kurtosis. For samples with positive values it turns out: $0 \le \tau < 1$ e $|\tau_r| < 1$ for $r \ge 3$.

Hosking e Wallis [11, 12, 13] developed a regionalization procedure based on LM ratio, articulated in four steps:

1. Screening of the data using discordancy measure test;
2. Identification and test of homogeneous regions;
3. Choice of a regional frequency distribution;
4. Parameters estimation of the regional frequency distribution.

The first step consists in an inspection of the data, towards the aim to identify errors, inconsistencies, trends and outliers, through the statistic D, that identifies sites grossly discordant with the group as a whole.

Hosking et al. [13] qualify the site as potentially discordant if the D value, calculated for every historical series in a fixed region, is greater than a critical value estimated by the authors at significance level equal to 10% (discordancy measure D is however significant only for regions with at least seven sites). For regions with at least 15 sites, a site is considered discordant if $D \geq 3$.

The second step consists in grouping sites towards the aim to identify a region with the support of classification techniques or multivariate statistical procedure (as cluster analysis) based on geographic, physical and climatic station characteristics. To investigate the regional homogeneity, Hosking proposed to evaluate the heterogeneity measure H_1, defined as [13]:

$$H_1 = \frac{V_1 - \mu_V}{\sigma_V} \qquad (6)$$

where V_1 is the weighted standard deviation of the at-site sample L-CV:

$$V_1 = \left\{ \frac{\sum_{i=1}^{N} n_i (\tau^i - \tau^R)^2}{\sum_{i=1}^{N} n_i} \right\}^{1/2} \qquad (7)$$

and μ_V and σ_V are respectively average and standard deviation of V_1, computed by simulating 500 homogeneous regions. These generated regions contain sites with the same record lengths of the region studied and the kappa distribution as parent distribution.

Hosking and Wallis declare a region "acceptably homogeneous" if $H_1<1$, "possibly heterogeneous" if $1 \leq H_1 < 2$ and "definitely heterogeneous" if $H_1 \geq 2$.

However the authors underline that a moderated heterogeneity ($1 \leq H_1 < 2$) yields a quantile estimation much more accurate than the at-site estimation; moreover they recommend H=2 "*as the point at which redefining … omissis … the region is very likely to be beneficial*" [13].

Furthermore Hosking, analogously to H_1, defines two other statistics for testing homogeneous regions: H_2 and H_3. The first one is based on L-CV and L-skewness and the last one on L-skewness and L-kurtosis.

In particular, the use of H_3 is suggested in hierarchical procedure of regionalization [13].

The third step consists in the choice of an appropriate frequency distribution for a homogeneous regions previously identified. Towards this aim, Hosking and Wallis define the statistic test Z, which allows to find the frequency distribution fitting among the well-known three parameters distributions:

Generalized Extreme Value (GEV), Generalized LOgistic (GLO), Generalized PAreto (GPA), LogNormal III (LN3) and PEarson III (PE3). The distribution is considered as a good fit of the observed data if $Z \leq |1{,}64|$ [13].

In the last step, the frequency distribution parameters are estimated basing on regional sample L-moments ratio.

Hosking et al. furnished the relations between LM and the mostly common frequency distribution parameters [11, 13].

2.3 *MGs model*

The principal hypothesis of index flood method is the invariance of the moment greater than the first order of the normalized variable, within a region considered statistically homogeneous.

Maione et al. [2, 16, 17], regarding annual maximum flood series, observe that the coefficients of variation CV and of skewness γ vary within large Italian areas considered homogeneous in TCEV hierarchical procedures. Moreover, the authors detected a link between these two statistics.

The latter remark suggested the use of parametric methods, which take into account the CV spatial variability and indirectly the γ spatial variability by means of the relationship γ(CV). Maione et al. formulated the two-parameters regional probabilistic model, called MG, depending on both the average, μ, and the coefficient of variation, CV:

$$X/\mu = f(T, CV) \qquad (8)$$

Following this approach, the model varies in the space according to CV; the X estimation depends on CV and on the scale factor, μ.

Towards the aim to further reduce the parameters to be estimated, the same authors proposed the simplified MGs model. The latter is based on the observation that the normalization of the variable X respect to the standard deviation σ, made the corresponding quantile not very sensitive to CV and γ changes. In this case, the quantile X/σ could be expressed as function of the only return period, T:

$$X/\sigma = f(T) \qquad (9)$$

As a consequence, the estimation of the variable X depends only on the scale parameter σ. In (9) we find again the index flood expression, with index value σ.

The authors derived the MGs model equation empirically, investigating the annual maximum floods observed in 249 stations placed in all Italy [16, 17].

Assuming that:

- the i_th value Q_{ij} of the generic historical series j is independent from the other values in the same series

and observing that:

- the values Q_{ij}/σ_j (normalized respect to the standard deviation σ_j) can be drawn from a single population,

the probability distribution P of the variable $\hat{Q}_j/\sigma_j$, where $\hat{Q}_j$ is the historical series maximum value, can be expressed by:

$$P_{\hat{Q}/\sigma}(q) = P_{Q/\sigma}(q)^{Nmed} \qquad (10)$$

where N_{med} indicates the historical observation series mean and q is the quantile.

Ordering the sample of N_{el} values $\hat{Q}_j/\sigma_j$ (N_{el} represent the number of historical series) in decreasing order, the quantile value, q and consequently the generic variable, Q/σ, is the one corresponding to the place element, N_{theor} [2]:

$$N_{theor}(q) = N_{el}[1 - P_{\hat{Q}/\sigma}(q)] = N_{el}[1 - P_{Q/\sigma}(q)^{N_{med}}] = \\ = N_{el}[1-(1-1/T(q))^{N_{med}}] \qquad (11)$$

So, known N_{el} and N_{med}, for each N_{theor}, and thus for a fixed $\hat{Q}_j/\sigma_j$, it is possible to evaluate the correspondent return period $T(\hat{Q}_j/\sigma_j)$ by equation (11).

The pairs (lnT, $\hat{Q}_j/\sigma_j$), representative of Italian rivers maximum flood studied, plotted in a semi-logarithmic diagram, showed, for T = 30-800 years, the following linear relationship [2]:

$$\frac{Q}{\sigma} = c + b \ln T \qquad (12)$$

where c and b are growth law parameters, valid in the whole Italian peninsula.

3. Application to Sicilian rainfall data

3.1 *Data*

The data used in this work are the rainfall annual maximum series of duration 1, 3, 6, 12 and 24 hours in 235 sites, placed in all Sicily. The observed period is 1928-1998 and every site had record length more than 10, with sample size mean equal to 29.2. The comparison between record length in VAPI study (1928-1981) and the present upgrade, is showed in figure 1.

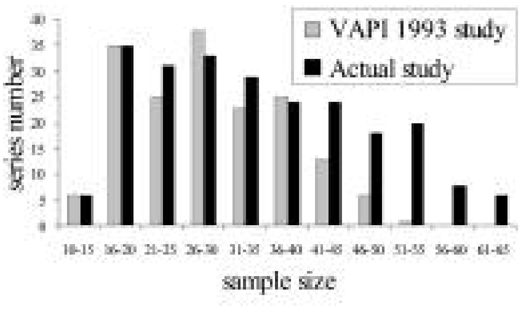

Fig. 1 - Comparison between record length in VAPI study (1928-1981) and the present upgrade (1928-1998).

3.2 *TCEV model*

Before investigating the first level of regionalization, the third moment order independence (coefficient of skewness γ or L-skewness τ_3) by duration was verified, for sample size n≥30. In figure 2 the γ and τ_3 substantial invariance by duration t is showed, where γ and τ_3 are respectively the means of coefficient of skewness and coefficient of L-skewness, sample size weighted. This observation implies the invariance of Θ^* and Λ^* by duration (temporal independence).

At the first level of regionalization, Sicily was hypothesized as a spatial homogeneous region. This hypothesis was verified through the heterogeneity meas-

Parent	Duration t [hours]				
Kappa	1	3	6	12	24
H_3 test	0.10	1.07	-0.08	-1.05	0.14

TABLE 1 - H_3 test for duration 1÷24 hours.

ure H_3, introduced by Hosking [13], for each duration. The results of the latter test (table 1), show the homogeneity of all island in τ_3 and τ_4 for all duration, excepting t=3 hours.

The value H_3=1.07 for t=3 hours indicates a potentially heterogeneity. However, in order to obtain a better quantile estimation, Hosking suggests to subdivide a region in sub-regions only if H>2. For this reason, the whole Sicily is also considered homogeneous for t=3 hours.

After checking the spatial independence of the third moment order, the values Θ^* e Λ^* were estimated for sites with n≥30 (record length mean equal to 48) and for all durations with the maximum likelihood method [5]. The resulting parameters are:

$$\Theta^* = 2.399 \text{ and } \Lambda^* = 0.360 \qquad (13)$$

At the second level of regionalization the spatial homogeneous sub-region individuation was investigated by observing the L-CV_1 mean spatial-temporal

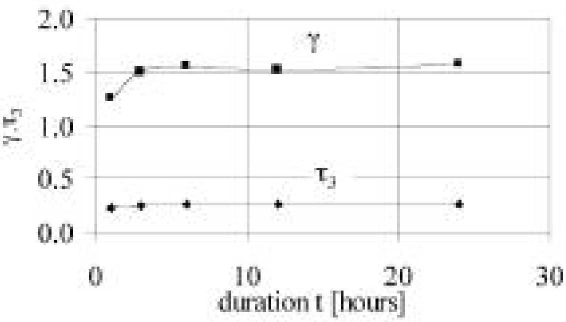

Fig. 2 - γ and τ_3 empirical means, sample size weighted, for t=1÷24 hours and n≥30.

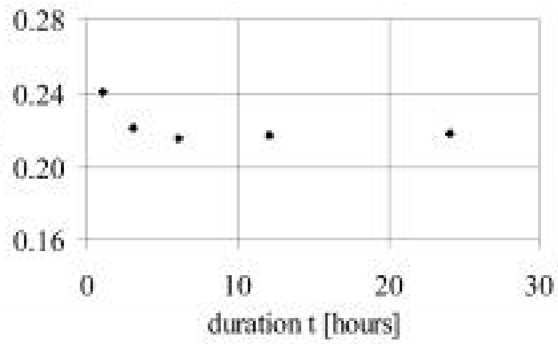

Fig. 3 - L-CV_1 mean, sample size weighted, for t=1÷24 hours and n≥20.

invariance, where $L\text{-}CV_1$ is the L-CV of ordinary component. The $L\text{-}CV_1$ value was sample size weighted, for n≥20 [9]. Regarding temporal dependence figure 3 suggested a different behavior for t=3÷24 hours and for t=1 hour.

This evidence suggested to subdivide the whole sample into two different data sets, one for t=1 hour and another one for t=3÷24 hours. For each data set the spatial homogeneous sub-region individuation was detected using the “k-means” method. This classification method, like the one-way analysis of variance, subdivides a set of objects in a fixed number of groups, maximizing the standard deviation between groups rather than inside groups.

Each site was characterized by its own $L\text{-}CV_1$ and U.T.M. coordinates, normalized on its own range.

For both t=1 hour and t=3÷24 hours sample, the analysis let to find two sub-regions, not coincident. To validate these classifications the not-parametric Kruskal-Wallis test was applied to $L\text{-}CV_1$ samples of the two sub-regions individuated for both t=1 hour and t=3÷24 hours.

The test, that verifies the null hypothesis H_0 of identical $L\text{-}CV_1$ distributions against the alternative hypothesis H_1 of different $L\text{-}CV_1$ distributions, showed the following results (table 2):

- for t=1 hour the hypothesis H_0 (sample $L\text{-}CV_1$ identical distribution) was accepted (p-value =0.393), indicating a single sub-region;
- for t=3÷24 hours the hypothesis H_0 (sample $L\text{-}CV_1$ identical distribution) was rejected (p-value <0.001) indicating two different sub-regions (figure 4).

Duration [hours]	p-value	Number of identified sub-regions
1	0.393	1
3, 6, 12, 24	<0.001	2

TABLE 2 - p-value related to Mann-Whitney test.

From this statistic evidence the following homogeneous sub-regions in $L\text{-}CV_1$ were individuated:

- one sub-region coincident with the whole Sicily, for t=1 hour;
- two sub-regions, called sub-region 1 and sub-region 2, for t=3÷24 hours (figure 4).

The regional parameter λ_1 was calculated by $L\text{-}CV_1$ spatial mean, inverting the following theoretical relationship:

$$L - CV_1 = \frac{0.312}{0.251 + \log \lambda_1} \qquad (14)$$

For each duration and sub-region, λ_1 results as follows:

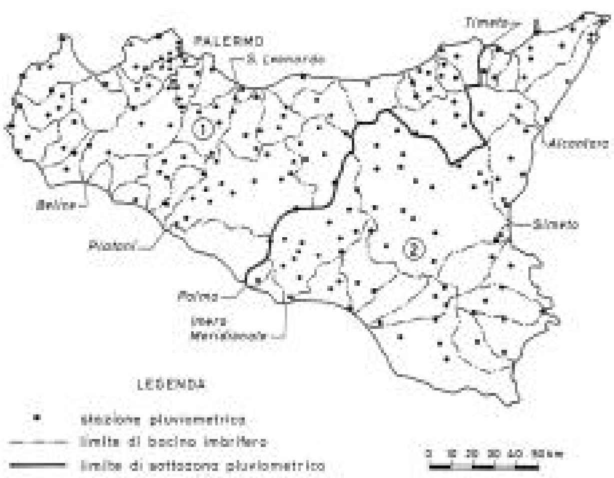

Fig. 4 - Sub-regions 1 and 2, homogeneus in $L\text{-}CV_1$, for t=3÷24 hours.

- t = 1 hour:
  λ_1 = 14.65 (all Sicily)
- t = 3÷24 hours:
  λ_1 = 24.57 (sub-region 1)
  λ_1 = 18.03 (sub-region 2)

Now, considering that $F = \frac{T-1}{T}$, the growth curve obtained inverting the equation (3), are:

- t = 1 hour:
  $$x(T)=0.256+1.306 \log T \qquad (15')$$
- t = 3÷24 hours:
  Sub-region 1:
  $$x(T)=0.309+1.174 \log T \qquad (15'')$$
  Sub-region 2:
  $$x(T)=0.278+1.250 \log T \qquad (15''')$$

The equations (15)[2] gives a quantile estimation slightly greater than the last pluviometric Sicilian study [5] for each duration t (+2 ÷10%).

At the third level of regionalization, it was necessary to estimate the index value, that, in this case, was the theoretical TCEV law mean, μ. By the substantial equality of empirical average m_c and μ, m_c was assumed as the index value.

To calculate m_c at ungaged sites or at sites with short sample size, under the hypothesis of the well-known pluviometric probability curve validity:

$$m_c=at^n \qquad (16)$$

it was sufficient to know the parameters a and n. These parameters, estimated for all 235 sites, were

[2] In the applications, for 1<t<3 hours, x(T) can be estimated by linear interpolation between x(T) from eq. (15’) and x(T) from eq. (15’’) if the site falls in sub-region 1 and between eq. (15’) and eq. (15’’’) if the site falls in sub-region 2.

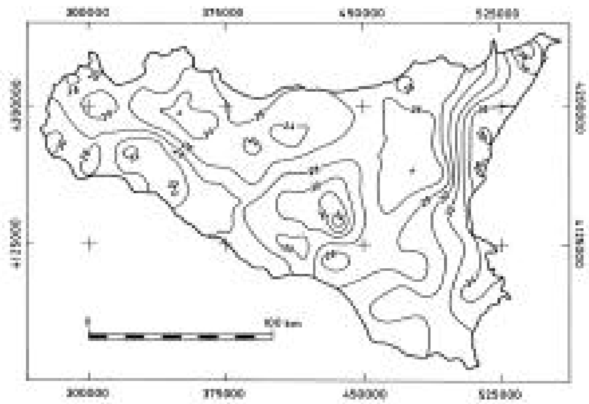

Fig. 5 - Iso-a map (equation (16)).

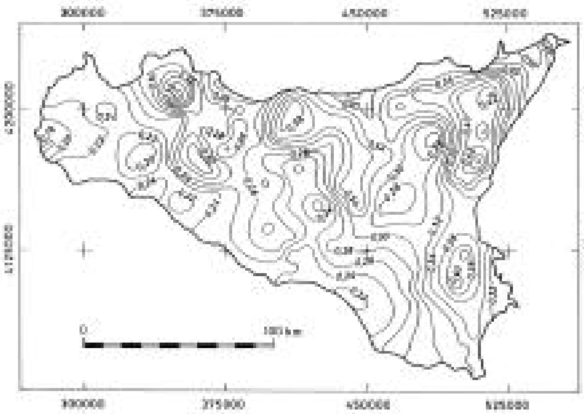

Fig. 6 - Iso-n map (equation (16)).

spatially interpolated through the exponential kriging method. In figure 5 and 6 the iso-a and iso-n curve for Sicily are showed.

The m_c estimation with iso-a and iso-n maps involves an error. To evaluate this error, the normalized error (*ne*) of m_c was used:

$$ne(m_c)=(\hat{m}_c - \dot{m}_c)/\dot{m}_c \qquad (17)$$

where $\hat{m}_c$ is calculated from the equation (16) with a and n obtained by maps in figures 5 and 6 and $\dot{m}_c$ is the historical series mean. In table 3, for each t, mean and standard deviation of $(ne)m_c$, valuated for 235 sites, are showed. In the same table it is possible to observe that $\hat{m}_c$ slightly overestimates $\dot{m}_c$.

	Duration t [hours]				
	1	3	6	12	24
$\langle ne(m_c) \rangle$ [%]	2.24	1.37	1.26	1.32	3.33
s.d. $[ne(m_c)]$ [%]	13.80	13.41	13.94	14.84	16.03

TABLE 3 - Mean and standard deviation (s.d.) of $ne(m_c)$, where $ne(m_c)$ is the normalized error of m_c, obtained by using iso-a and iso-n maps and for duration t=1÷24 hours.

3.3 *LM model*

In order to obtain a good estimation of the L-moments ratio τ_3 and τ_4, the samples with n≥30 were considered. As a consequence the sites analyzed were 109 in the whole Sicily, with a sample size mean equal to 41.2.

The discordancy measure identified only 5 samples with D≥3 for each duration t. The analysis of these discordant series showed the presence of outliers in these sites; thus, all stations were considered in the analysis.

In the second step of the procedure, the test H_1 was used to verify the homogeneity of the Sicilian region.

The results of the H_1 statistic, showed in table 4, indicated a region acceptably homogeneous for t=1 hour and possibly heterogeneous for t=3÷24 hours.

As already mentioned a moderated heterogeneity ($1 \leq H_1 < 2$) yields a quantile estimation much more accurate than the at-site estimation and only for H>2 it is convenient to redefine the region studied [13]. For these reasons Sicily was considered a unique homogeneous region for all durations.

Moreover, the value of H_1=-0.30 for t=1 hour (table 4) indicates a positive correlation between the data values at different sites.

	Duration t [hours]				
	1	3	6	12	24
H_1 test	-0.30	1.97	1.91	1.96	1.82

TABLE 4 - H_1 test for duration 1÷24 hours.

The choice of a distribution for the whole Sicily and for each duration was previously made by using the L-moments ratio diagram (τ_3,τ_4).

Figure 7 shows the pairs (τ_3^R, τ_4^R) for each duration, where τ_3^R and τ_4^R are respectively the τ_3 and τ_4 regional weighted mean of the observed data, and the $\tau_4(\tau_3)$ of theoretical probability distributions GEV,

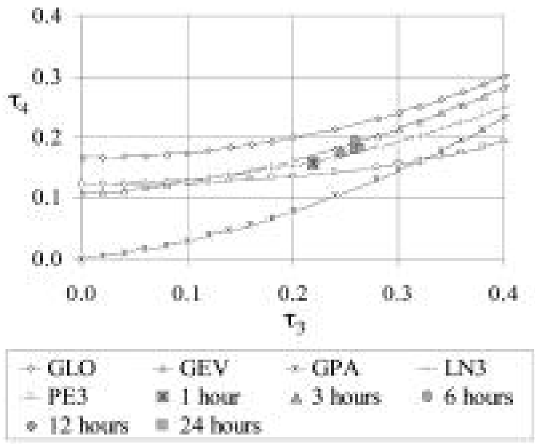

Fig. 7 - L-moments ratio diagram (τ_3,τ_4).

	Duration t [hours]				
	1	3	6	12	24
GLO	7.13	4.21	3.29	5.35	4.91
GEV	2.03	**0.21**	**-0.63**	**1.04**	**1.16**
GPA	-9.85	-9.60	-10.28	-9.34	-8.10
LN3	**0.43**	-1.81	-2.68	**-0.85**	**-0.88**
PE3	-2.65	-5.42	-6.30	-4.27	-4.49

TABLE 5 - Z test for GEV, GLO, GPA, LN3 and PE3 distribution, for duration 1÷24 hours.

GLO, GPA, LN3 e PE3. The closeness of the regional pairs (τ_3^R, τ_4^R) to a given distribution gives a visual indication of the distribution expected to have a good fit of the sample. Looking figure 7, GEV and LN3 distribution are expected to have a good fit of the observed data.

This indication was validated by the test Z [13]. The results, reported in table 5, indicate the acceptation of GEV distribution for t=3÷24 hours and LN3 for t=1, 12, 24 hours (bold character in table 5), as previously showed in the L-moments ratio diagram (τ_3,τ_4).

Considering that GEV distribution provided the best fit of the data for t=3÷24 hours and for t=1 hours the Z value (2.03) was not so far from the critical value 1.64, the GEV law was chosen as the probability distribution for each duration.

The GEV distribution expression is:

$$F(x) = \exp\{-[1-k(x-\xi)/\alpha]^{1/k}\} \text{ for } k\neq 0 \qquad (18)$$

where ξ is the location parameter, α is the scale parameter and k is the shape parameter. For the estimation of the parameters ξ, α and k Hosking developed the following approximation [13]:

$$\begin{aligned} k &= 7.8590c + 2.9554c^2 \\ \alpha &= \lambda_2 k / \{\Gamma(1+k)(1-2^{-k})\} \\ \xi &= \lambda_1 - \alpha\{1-\Gamma(1+k)\}/k \\ c &= \frac{2}{3+\tau_3} - \frac{\ln 2}{\ln 3} \end{aligned} \qquad (19)$$

where λ_1, λ_2 and τ_3 are the weighted regional LM.

The estimated parameters are reported in table 6.

For t=1 hour k was near to 0, indicating, for this duration, a good fit to the Gumbel distribution.

	Duration t [hours]				
Parameters	1	3	6	12	24
ε	0.784	0.780	0.784	0.789	0.774
α	0.329	0.303	0.296	0.300	0.306
k	-0.076	-0.132	-0.135	-0.114	-0.141

TABLE 6 - GEV distribution parameters for duration 1÷24 hours.

Furthermore it was observed that, except for t=1 hour, the parameters α and k were approximately constant. Therefore a unique GEV distribution for t=3÷24 hours was adopted, with ξ, α and k equal to the average of the values ξ_i, α_i and k_i for i=3, 6, 12, 24 hours.

The expression of the GEV quantile x(F) is:

$$x(F) = \xi + \frac{\alpha}{k}\left\{1-(-\ln F)^k\right\} \text{ for } k\neq 0 \qquad (20)$$

where $F = \frac{T-1}{T}$.

Thus, the growth curves x(T), called GEV-LM, are:

- t = 1 hour:

$$x(T) = 0.784 - 4.347\left\{1-\left[-\ln\left(\frac{T-1}{T}\right)\right]^{-0.076}\right\} \qquad (21')$$

- t = 3÷24 hours:

$$x(T) = 0.782 - 2.308\left\{1-\left[-\ln\left(\frac{T-1}{T}\right)\right]^{-0.131}\right\} \qquad (21'')$$

The estimation of the variable X(T) can be derived by equation (1), where the growth curve is expressed by (21') or (21''), respectively for t=1 hour and t=3÷24 hours, and the index flood is the at-site rainfall mean. In ungaged sites the scale factor may be calculated with the equation (16), with a and n obtained by the iso-a and iso-n maps (figures 5 and 6).

3.4 *MGs model*

The link between γ and CV empirical series for all durations was observed for Sicily (figure 8), assuming the following expression:

$$\gamma = 4.58CV - 0.82 \qquad (22)$$

This observation confirmed the remark of Maione et al. [16, 17], justifying the adoption of their parametric model. The further observations that the values X_{Max}/σ, obtained by normalizing the maximum values of sample series, X_{Max}, respect to the standard deviation σ of its series, were independent by CV (figure 9), led to the simpler MGs parametric model.

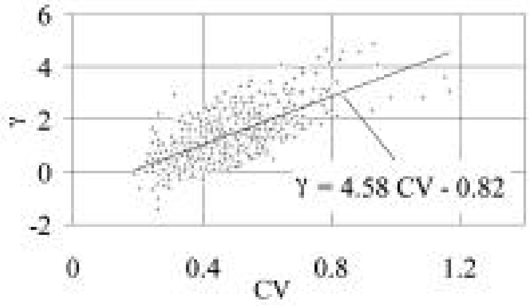

Fig. 8 - Relationship γ (CV) for historical rainfall series of duration t=1÷24 hours.

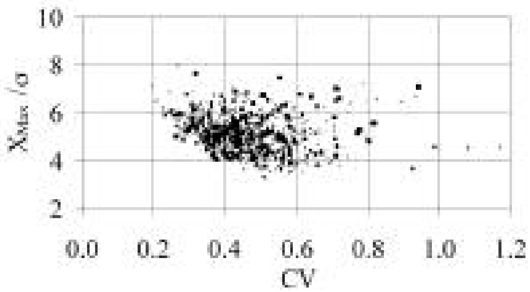

Fig. 9 - Dispersion of pairs (CV, X_{max}/σ) for duration t=1÷24 hours.

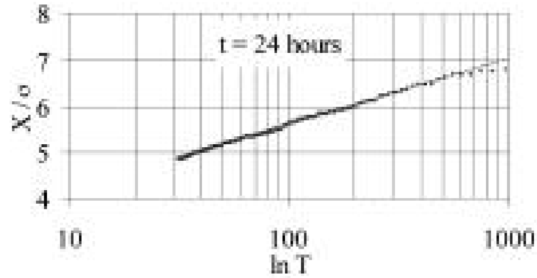

Fig. 10b - Dispersion of pairs (T, X/σ) for duration t=24 hours.

Following the procedure introduced by Maione et al. [16, 17], (par. 2.3) N_{med} is 29 and N_{el} is 235.

For range T=30÷900 years, the pairs (T, X_{Max}/σ), with T derived by (11), plotted in a semi-logarithmic diagram, indicate a good fit to the following linear equation (figures 10):

$$\frac{X}{\sigma} = c + b \ln T \tag{23}$$

where values c and b for duration t=1÷24 hours are reported in table 7. It was possible to observe that the parameters c and b are substantially constant for t=1÷12 hours range. Therefore, the constant values c=3.23 and b=0.509 were assumed for t=1÷12 hours, while it was maintained c=2.80 and b=0.614 for t=24 hours[3].

So the quantile function became, respectively:

for t=1÷12 hours

$$\frac{X_{t,T}}{\sigma_t} = 3.235 + 0.509 \ \ln T \tag{24'}$$

for t=24 hours

$$\frac{X_{t,T}}{\sigma_t} = 2.797 + 0.614 \ \ln T \tag{24''}$$

To verify the hypotheses that support the MGs model, a comparison between the X/σ normalized

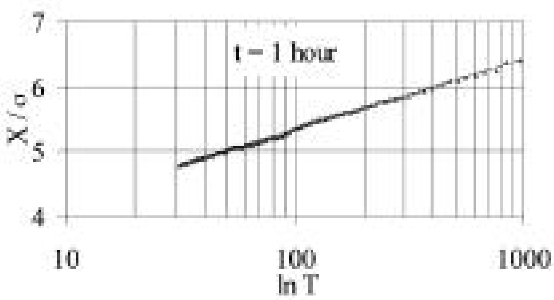

Fig. 10a - Dispersion of pairs (T, X/σ) for duration t=1 hour.

[3] In the applications, for 12<t<24 hours, x(T) can be estimated by linear interpolation between x(T) from eq. (24') and x(T) from eq. (24'').

Duration [hours]	c	b
1	3.167	0.472
3	3.237	0.512
6	3.328	0.512
12	3.209	0.538
24	2.797	0.614

TABLE 7 - c and b MGs parameters for duration t=1÷24 hours.

quantile, obtained by GEV (assumed as parent distribution), and the MGs model here derived, was established.

For T=200 years, in figure 13 are reported:

- the pairs (X/σ, CV), obtained by GEV with parameters computed satisfying the empirical link (10) (GEV points);
- the two MGs laws, expressed by equations (24), that obviously are horizontal straight lines.

Figure 11 shows that, assuming GEV law as parent distribution, the quantile X/σ is independent from CV and, according to equation (22), from γ. In addition, the MGs laws overlap GEV points representing the Sicilian extreme rainfall sample, showing a good fit to Sicilian data.

To estimate $X_{t,T}$ with equations (24) in a generic site, it was necessary to know or to estimate the at-site σ_t value. To calculate σ_t at ungaged sites or at sites with short record, it is usual to recur to regression

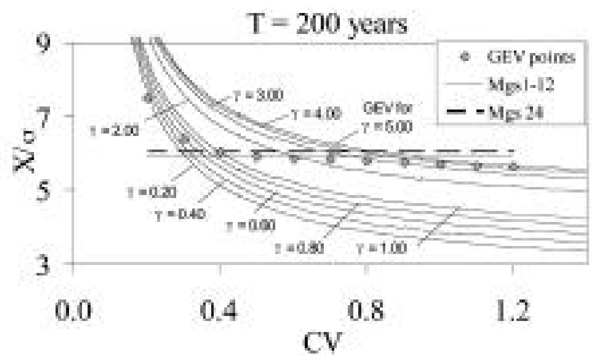

Fig. 11 - X/σ quantile obtained by GEV (GEV points) and MGs model quantile for t=1÷12 hours and t=24 hours.

analysis using measurable parameters such as altitude, longitude, latitude, distance of the site from the sea and so on. Cause weak links found between σ_t and geographic variables, we chose to interpolate the empirical value, $\dot{\sigma}_t$, obtained in the 235 sites studied, with exponential kriging technique and for all Sicily.

In figures 12 the five maps of iso-σ for t=1, 3, 6, 12 and 24 hours are shown.

The σ_t estimation through iso-σ maps involves an error, evaluated in normalized form as:

$$ne(\sigma_t) = (\hat{\sigma}_t - \dot{\sigma}_t)/\dot{\sigma}_t \qquad (25)$$

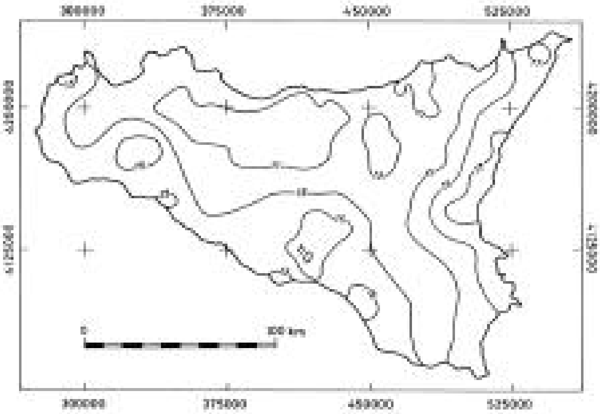

Fig. 12a - Iso-σ map for t=1 hour.

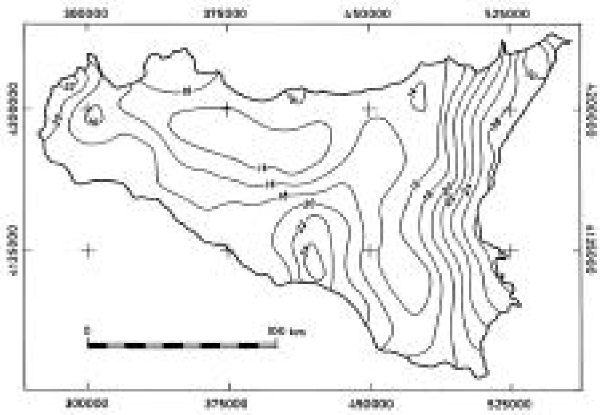

Fig. 12b - Iso-σ map for t=3 hours.

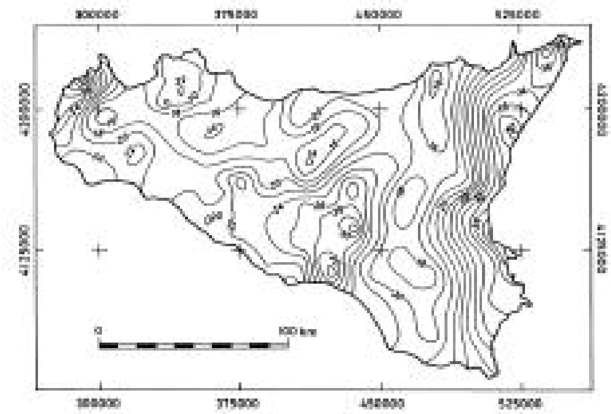

Fig. 12c - Iso-σ map for t=6 hours.

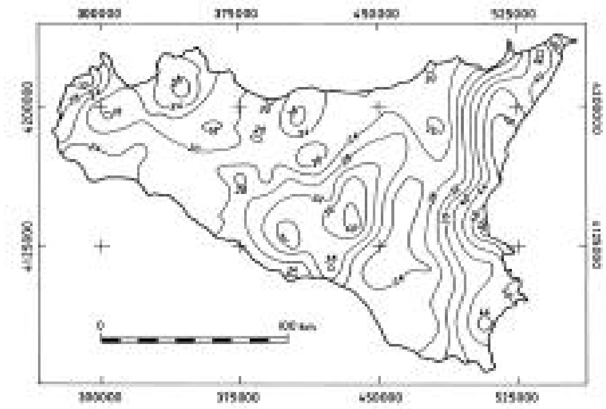

Fig. 12d - Iso-σ map for t=12 hours.

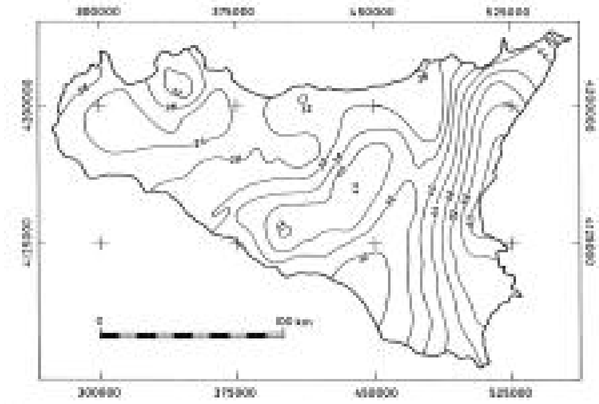

Fig. 12e - Iso-σ map for t=24 hours.

where ne is the normalized error of σ_t, $\hat{\sigma}_t$ is the standard deviation obtained by iso-σ maps and $\dot{\sigma}_t$ is the historical series standard deviation.

In table 8, for each t, mean and standard deviation of $ne(\sigma_t)$, evaluated for the 235 sites, are showed. Results show that $\hat{\sigma}_t$ slightly overestimates $\dot{\sigma}_t$(5÷11 %).

Comparison between tables 3 and 8 shows that mean and standard deviation of $ne(\sigma_t)$ are greater than mean and standard deviation of $ne(m_c)$. In other words, this comparison shows that the estimation of m_c by iso-a and iso-n maps is better than the estimation of σ by iso-σ maps. This evidence has an important effect on extreme rainfall estimation by regional models based on this two different index values.

	Duration t [hours]				
	1	3	6	12	24
$\langle ne(\sigma_t) \rangle$ [%]	5.71	8.04	9.49	10.04	11.04
s.d. $[ne(\sigma_t)]$ [%]	27.18	30.70	34.26	35.11	35.27

TABLE 8 - Mean and standard deviation (s.d.) of $ne(\sigma_t)$ where $ne(\sigma_t)$ is the normalized error of σ_t, obtained by using iso-σ maps and for duration t=1÷24 hours.

Coming back to figures 12, it was observed that the spatial distribution of σ shows the same behavior for every t: high and growing value in the oriental coast of Sicily and in a center-southern limited area. This fact suggested a scale behavior, expressed as follows:

$$\frac{\sigma_t}{d \cdot t^f} = \sigma_{t^*} \tag{26}$$

where t* is the fixed duration and $d \cdot t^f$ is the scale function, with coefficients d and f. After deriving t*=6 hours as the smaller standard deviation error in the range t=1÷24 hours, the parameter values were obtained: d=0.61 and f=0.29. Therefore, the (26) relationship becomes:

$$\sigma_t = 0.61\ t^{0.29}\ \sigma_6 \tag{27}$$

where σ_6 is the standard deviation for t=6 hours.

Thus, it is possible to estimate σ_t by using equation (27) and only the map of σ_6.

To evaluate the estimate error of (27), the following normalized error (*ne'*) of σ_t was used:

$$ne'(\sigma_t) = (\hat{\sigma}'_t - \dot{\sigma}_t)/\dot{\sigma}_t \tag{28}$$

where $\hat{\sigma}'_t$ is obtained by equation (27) and σ_6 map.

In table 9, for each t, $ne'(\sigma_t)$ mean and standard deviation, valuated for the 235 sites, are showed. The same table shows that $\hat{\sigma}'_t$ overestimates σ_t on average by 10%.

	Duration t [hours]				
	1	3	6	12	24
$\langle ne'(\sigma_t) \rangle$ [%]	10.73	10.56	9.49	12.40	11.69
s.d. $[ne'(\sigma_t)]$ [%]	32.32	31.89	34.26	38.11	39.59

TABLE 9 - Mean and standard deviation (s.d.) of $ne'(\sigma_t)$, where $ne'(\sigma_t)$ is the normalized error of σ_t, obtained by using iso-σ_6 map for t=6 hours and equation (27).

4. Regional models comparison

In order to establish the best predictive regional model, the TCEV, GEV-LM and MGs regional estimations were compared with the at-site estimation, for the stations with record length n≥45. The return period T was chosen equal to 200 years[4] and the local estimate was computed by GEV and Gumbel distribution with parameters obtained by weighted moment method [16, 17].

The comparison was carried out examining, for the regional models, the index value estimation in the two following cases:

- Case 1: Index value estimated by historical series;
- Case 2: Index value estimated by regional interpolated maps.

The first case explains the regional distribution performance and the second one indicates the regional model performance taking into account the index value estimation error.

The regional models performance is described by average (μ_{ne}), standard deviation (σ_{ne}), maximum positive ($maxp_{ne}$) and maximum negative ($maxn_{ne}$) of normalized error:

$$\frac{X_{reg} - X_{loc}}{X_{loc}} \tag{29}$$

where X_{reg} is the precipitation obtained by regional models and X_{loc} is the precipitation obtained by local models (table 10).

Figures 13 and 14 show average (or bias) and standard deviation histograms of normalized error.

Results show, in Case 1, a lower μ_{ne} and σ_{ne} using MGs model than TCEV or GEV model, for each duration.

In Case 2, results show, for t=1÷12 hours, a lower μ_{ne} using MGs model than TCEV or GEV model, but, for t=24 hours, the three regional models perform analogously. Instead, the σ_{ne} is similar for the three regional models studied, with a little advantage for MGs model for t=1÷12 hours.

Comparing Case 1 and Case 2 we can observe that the index value estimation error have an important effect on the extreme rainfall estimation. In fact, changing from Case 1 to Case 2, it is possible to notice that MGs model σ_{ne} grows up approaching the TCEV and GEV model σ_{ne}. As already told, this evidence is explained by the observation that σ_t estimate error is greater than m_c estimate error.

After all, we suggest the use of MGs model for t=1÷12 hours, cause a lower μ_{ne} and a little advantage in term of σ_{ne} using the latter model than TCEV or GEV model.

Instead for t=24 hours, the use of MGs, TCEV and GEV regional models gives the same result.

Furthermore, we observe the same behavior for TCEV and GEV-LM models, also detected by Brath in center-northern Italy [4].

5. Conclusions

The present study concerned the regional frequency analysis of extreme rainfall in Sicily. In particular, the TCEV regional model, proposed by Rossi et al. [20] applied in Sicily in the VAPI Project (CNR), was updated. The update allowed:

- to identify a single growth curve for all Sicily, for t=1 hour;
- to identify the growth curves for two sub-regions, for t=3÷24 hours.

The regional model based on linear moments [13]

[4] No evident trend was detected in Sicily for annual high intensity rainfall [18].

		GUMBEL (local estimate)						GEV_{loc} (local estimate)					
		GEV-LM		TCEV		MGs		GEV-LM		TCEV		MGs	
		CASE 1	CASE 2	CASE 1	CASE 2	CASE 1	CASE 2	CASE 1	CASE 2	CASE 1	CASE 2	CASE 1	CASE 2
1 hour	μ_{ne} (%)	9	11	22	24	1	2	10	11	22	24	0	2
	σ_{ne} (%)	10	15	11	17	5	14	16	21	18	23	5	19
	$maxp_{ne}$ (%)	34	48	49	65	15	36	45	60	62	78	11	47
	$maxn_{ne}$ (%)	-17	-9	-7	1	-13	-19	-29	-19	-21	-10	-12	-23
3 hours	μ_{ne} (%)	17	15	17	16	0	2	19	18	19	18	2	4
	σ_{ne} (%)	12	18	12	18	6	15	16	20	16	20	6	17
	$maxp_{ne}$ (%)	42	66	39	62	15	40	56	71	59	67	15	44
	$maxn_{ne}$ (%)	-13	-23	-11	-21	-13	-23	-13	-23	-11	-21	-12	-23
6 hours	μ_{ne} (%)	15	12	15	12	1	-2	12	8	12	9	-3	-5
	σ_{ne} (%)	14	21	14	21	7	17	21	26	21	26	5	19
	$maxp_{ne}$ (%)	41	60	44	64	20	38	52	72	55	76	6	48
	$maxn_{ne}$ (%)	-21	-32	-23	-34	-13	-42	-32	-42	-34	-43	-14	-50
12 hours	μ_{ne} (%)	14	9	14	10	1	-3	16	11	16	11	3	-2
	σ_{ne} (%)	15	23	14	23	8	21	17	24	16	24	7	21
	$maxp_{ne}$ (%)	44	61	41	62	19	45	45	62	48	64	19	53
	$maxn_{ne}$ (%)	-19	-30	-21	-31	-14	-43	-19	-30	-21	-31	-14	-43
24 hours	μ_{ne} (%)	10	6	10	6	6	-3	7	3	7	3	1	-6
	σ_{ne} (%)	15	24	14	23	8	24	22	29	21	28	5	27
	$maxp_{ne}$ (%)	49	68	46	64	28	47	57	75	54	71	12	46
	$maxn_{ne}$ (%)	-31	-34	-29	-33	-15	-43	-44	-46	-43	-45	-11	-52

TABLE 10 - Normalized error statistics of (X_{reg}–X_{loc}), where X_{reg} and X_{loc} are respectively the regional and local estimates, for duration t=1÷24 hours, T=200 years, sites with n≥45 and Case 1 and Case 2.

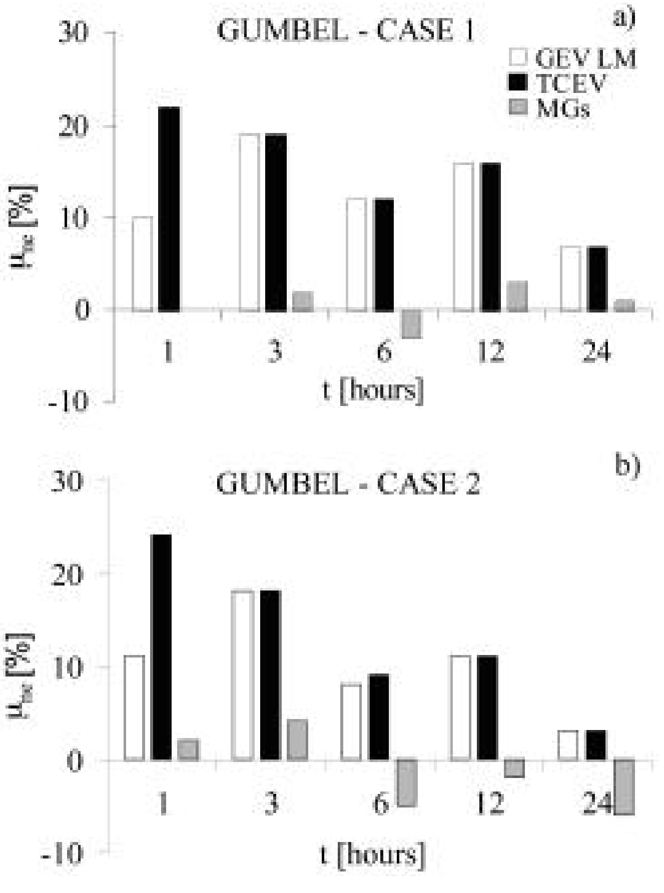

Fig. 13 - Comparison in term of μ_{ne} between regional models (GEV-LM, TCEV, MGs) and GEV local model for Case 1 (figure 13a) and Case 2 (figure 13b) and for t=1÷24 hours.

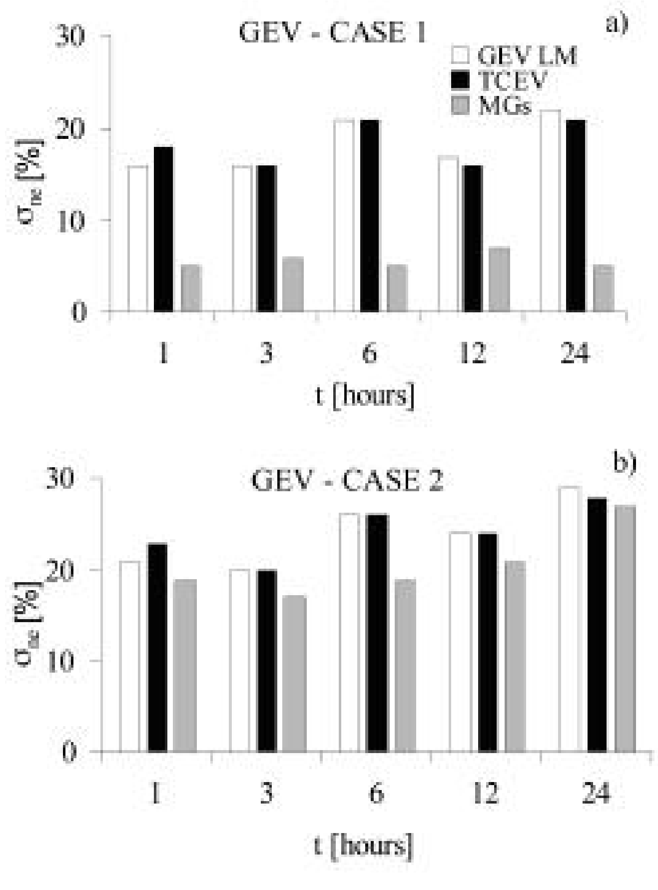

Fig. 14 - Comparison in term of σ_{ne} between regional models (GEV-LM, TCEV, MGs) and GEV local model for Case 1 (figure 14a) and Case 2 (figure 14b) and for t=1÷24 hours.

allowed the identification of the GEV as the regional probability distribution and the individuation of two growth curves valid in all Sicily dependent on duration t: one for t=1 hour and another one for t=3÷24 hours.

The adoption of the regional parametric model introduced by Maione et al. [16, 17] allowed to identify a growth curve for t=1÷12 hours and another one for t=24 hours, valid in all Sicily.

At gauged sites the index value (mean of the pluviometric variable for TCEV and GEV regional model and standard deviation of the pluviometric variable for MGs model) can be estimated by historical series, while, in ungaged site, the index term can be evaluated by using interpolated maps.

The three model regional estimations were compared with the at-site estimation, computed by GEV and Gumbel distribution, for the stations with record length n≥45 and return period T =200 years.

The comparison was carried out taking into consideration, for the regional models, the index value estimation in the two following cases:

- Case 1: Index value estimated by historical series;
- Case 2: Index value estimated by regional maps.

The results indicate that, in Case 1, the MGs regional model performs better than the other two regional models for each duration t but, in Case 2, the performance of MGs model is slightly better than the other regional models studied only for t=1÷12 hours.

Furthermore, we observe the same behavior for TCEV and GEV-LM models, also detected by Brath in center-northern Italy [4].

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Summary

The study regarded the regional frequency analysis of extreme rainfall in Sicily, using rainfall annual maximum series of duration 1, 3, 6, 12 and 24 hours in 235 sites, stationed in all Sicily and for the period 1928-1998.

The applied regional models, after the appropriate verifications, were: the TCEV hierarchical model (Rossi et al.), the model based on the use of the linear moments LM (Hosking et al.) and the MGs parametric model (Maione et al.).

For ungaged sites, as in a flood index approach, the index value of each model was evaluated through maps derived from spatial interpolation.

The three models regional estimations were compared with the at-site estimation, computed by GEV and Gumbel distribution, for the stations with record length n≥45 and return period T =200 years.

The comparison was carried out taking into consideration, for the regional models, the index value estimation in the two following cases:

- Case 1: Index value estimated by historical series;
- Case 2: Index value estimated by interpolated maps.

The results indicate that, in Case 1, the MGs regional model performs better than the other two regional models for each duration t but, in Case 2, the performance of MGs model is weakly better than the other regional models studied only for t=1÷12 hours.

Furthermore, we observe the same behavior of TCEV and GEV-LM models, also detected by Brath in center-northern Italy [4].

Key words: rainfall, regional model, MGs, TCEV, L-moments.

Appendix
Example of extreme rainfall calculation

In this appendix it is estimated the extreme rainfall value $X_{t,T}$ with the three regional models illustrated in this paper in an ungaged site called “P” (U.T.M. coordinates: Lon East = 375.000 m and Lat North = 4.200.000 m,) for durations t_1=4.5 hours and t_2=18 hours and return period T=50 years.

The three regional estimates are compared with the local estimations, computed by using Gumbel distribution, in the nearby Ciminna station (UTM coordinates: Lon East= 373.384 m and Lat North = 4.194.980 m).

<u>TCEV model</u>

Falling P in sub-region 1 (figure 4) and being t≥3, the growth factor x(T) is (eq. 15”):

$x(T) = 0.309 + 1.174 \log T =$
$= 0.309 + 1.174 \log (50) = 2.30$

by maps in figures 5 and 6:

$a = 26$ mm ; $n = 0.28$

t_1=4.5 hours

$m_c = a\, t^n = 26 * 4.5^{0,28} = 39.62$ mm

$X_{t=4.5,T=50} = x(T) * m_c = 2.30 * 39.62 = 91.13$ mm

t_2=18 hours

$m_c = a\, t^n = 26 * 18^{0,28} = 58.40$ mm

$X_{t=18,T=50} = x(T) * m_c = 2.30 * 58.40 = 134.32$ mm

<u>GEV-LM model</u>

t_1=4.5 hours

Because t≥3, the growth factor x(T) is (eq. 21”):

$$x(T) = 0.782 - 2.308\left\{1 - \left[-\ln\left(\frac{T-1}{T}\right)\right]^{-0.131}\right\} =$$
$$= 0.782 - 2.308\left\{1 - \left[-\ln\left(\frac{50-1}{50}\right)\right]^{-0.131}\right\} = 2.32$$

$m_c = a\, t^n = 26 * 4.5^{0,28} = 39.62$ mm

$X_{t=4.5,T=50} = x(T) * m_c = 2.32 * 39.62 = 91.92$ mm

t_2=18 hours

$m_c = a\, t^n = 26 * 18^{0,28} = 58.40$ mm

$X_{t=18,T=50} = x(T) * m_c = 2.32 * 58.40 = 135.49$ mm

<u>MGs model</u>

t_1=4.5 hours

$\sigma_{t=4.5}$ is evaluated by equation (27), where $\sigma_{t=6}$ is estimated by figure 12c:

$\sigma_6 = 15$ mm

$\sigma_{t=4.5} = 0.61\, t^{0.29}\, \sigma_6 = 0.61 * 4.5^{0.29} * 15 = 14.15$ mm

Because t≤12, it is used the equation (24’):

$$X_{t=4.5,T=50} = \sigma_{t=4.5}(3.235 + 0.509 \ln T) =$$
$$= 14.15 * (3.235 + 0.509 \ln(50)) = 74.00\text{mm}$$

t_2=18 hours

$\sigma_{t=18}$ is evaluated by equation (27), where $\sigma_{t=6}$ is estimated by figure 12c:

$\sigma_6 = 15$ mm

$\sigma_{t=18} = 0.61\, t^{0.29}\, \sigma_6 = 0.61 * 18^{0.29} * 15 = 21.16$ mm

Because 12<t<24, $X_{t,T}$ is evaluated by linear interpolation between eq. (24’) and eq. (24”):

$$X_{t=18,T=50} = \sigma_{t=18}\left[\frac{1}{2}(3.235 + 0.509 \ln(50)) + \right.$$
$$\left. + \frac{1}{2}(2.797 + 0.614 \ln(50))\right] = 110.24\text{mm}$$

<u>Local estimate at Ciminna Station</u>

By Gumbel law application to Ciminna station extreme rainfall data (duration t = 1, 3, 6, 12, 24 hours; record length equal to 48), $X_{t,T}$ results:

$X_{t=4.5,T=50} = 72.16$ mm

$X_{t=18,T=50} = 115.60$ mm

By comparing this local estimation against regional ones it is noticed that MGs model performs better than TCEV or GEV-LM model. Furthermore, TCEV and GEV-LM model perform analogously.