

## **Appendix**

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### **A method for handlebars ballast calculation in order to reduce vibrations transmissibility in walk behind tractors**

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## I vibration mode - Horizontal deformation

$K_{h_{tor}} = \frac{F_h}{\gamma_{h_{tor}}}$  : horizontal bending stiffness of the whole structure,

$\gamma_{h_{tor}} = B_1 + B_2 + B_3 d' + (A_{13} d'^2 - A_{14} d'^3) \cos(\gamma - \theta)$  : lateral displacement of the handles

horizontal ends,

$$\gamma_{h_1} = \theta x + [A_5 x^2 - A_6 x^3] \cos \theta \quad x \in [0, b],$$

$$\gamma_{h_2} = B_1 + [A_9 s^2 - A_{10} s^3] \cos \theta + \theta s + [A_3 b - A_4 s^2] s \quad s \in [0, c],$$

$$\gamma_{h_3} = B_1 + B_2 + B_3 t + [A_{13} t^2 - A_{14} t^3] \cos(\gamma - \theta) \quad t \in [0, d'],$$

$$\theta = [A_1 a - A_2 a^2], \quad B_1 = \theta b + [A_5 b^2 - A_6 b^3] \cos \theta, \quad B_2 = [A_9 c^2 - A_{10} c^3] \cos \theta + [\theta + A_3 b - A_4 b^2] c,$$

$$B_3 = [\theta + A_3 b - A_4 b^2 + A_7 c - A_8 c^2], \quad A_1 = \frac{F_h}{G_1 I_1} \left[ (c + d) \cos \beta + \frac{b}{\sqrt{2}} + a \sin \alpha \right], \quad A_2 = \frac{F_h}{2 G_1 I_1} \sin \alpha,$$

$$A_3 = \frac{F_h}{E_2 I_2} (c + d + b), \quad A_4 = \frac{F_h}{2 E_2 I_2}, \quad A_5 = \frac{F_h}{2 E_2 I_2} (c + d + b), \quad A_6 = \frac{F_h}{6 E_2 I_2}, \quad A_7 = \frac{F_h}{E_3 I_3} (c + d),$$

$$A_8 = \frac{F_h}{2 E_3 I_3}, \quad A_9 = \frac{F_h}{2 E_3 I_3} (c + d), \quad A_{10} = \frac{F_h}{6 E_3 I_3}, \quad A_{11} = \frac{F_h}{2 E_a I_a} \cos \gamma d', \quad A_{12} = \frac{F_h}{4 E_a I_a} \cos \gamma,$$

$$A_{13} = \frac{F_h}{4 E_a I_a} \cos \gamma d', \quad A_{14} = \frac{F_h}{12 E_a I_a} \cos \gamma. I_1 = a \sin \alpha + \frac{b}{\sqrt{2}} + c \cos \beta,$$

$$I_2 = a \sin \alpha + \frac{b}{\sqrt{2}} + (c + d) \cos \beta,$$

$K_{h_p} = \frac{F_h}{[B_1 + (A_9 c^2 - A_{10} c^3) \cos \theta + \theta c + (A_3 b - A_4 b^2)]}$  : horizontal bending stiffness of the pillar.

$K_r = \frac{F_L L^2}{\Delta \gamma_L (F_L)}$  : torsional bending stiffness of the elastic silent-block.

## II vibration mode - Vertical deformation

$$K_{v_{tor}} = \frac{F_v}{J_{v_{tor}}} : \text{vertical bending stiffness of the whole structure,}$$

$\gamma_{v_{tor}} = B_6 + A_{23}d + A_{24}d^2 - A_{25}d^3$  : lateral displacement of the vertical ends of the handles,

$$\gamma_{v_1} = A_{15}x^2 - A_{16}x^3 \quad x \in [0, a], \quad \gamma_{v_2} = B_4 + A_{17}s - A_{18}s^2 - A_{19}s^3 \quad s \in [0, b],$$

$$\gamma_{v_3} = B_5 + A_{20}n + A_{21}n^2 - A_{22}n^3 \quad n \in [0, c], \quad \gamma_{v_4} = B_6 + A_{23}t + A_{24}t^2 - A_{25}t^3 \quad t \in [0, c],$$

$$B_4 = A_{15}a^2 - A_{16}a^3, \quad B_5 = B_4 + A_{17}b + A_{18}b^2 - A_{19}b^3, \quad B_6 = B_5 + A_{20}c + A_{21}c^2 - A_{22}c^3,$$

$$A_{15} = \frac{F_v}{2E_1 I_1} \left[ \sin^2 \alpha a + (c+d) \cos \beta \sin \alpha + \frac{b}{\sqrt{2}} \sin \alpha \right], \quad A_{16} = \frac{\sin^2 \alpha F_v}{6E_1 I_1},$$

$$A_{17} = \frac{F_v}{E_1 I_1} \left\{ \left[ \frac{\sin^2 \alpha a}{2} + \left( (c+d) \cos \beta + \frac{b}{\sqrt{2}} \right) \right] a \right\} \frac{1}{\sqrt{2}}, \quad A_{18} = \frac{F_v}{2\sqrt{2} E_2 I_2} \left[ (c+d) \cos \beta + \frac{b}{\sqrt{2}} \right],$$

$$A_{19} = \frac{F_v}{12E_2 I_2}, \quad A_{20} = \frac{F_v}{E_1 I_1} \left\{ \left[ \frac{\sin^2 \alpha a}{2} + \left( (c+d) \cos \beta + \frac{b}{\sqrt{2}} \right) \right] a \right\} \cos \beta + \frac{F_v}{E_2 I_2} \left[ (c+d) b \cos \beta \sin \alpha + \frac{b^2}{2\sqrt{2}} \right] \cos \beta,$$

$$A_{21} = \frac{F_v}{E_3 I_3} \left[ (c+d) \frac{\cos^2 \beta}{2} \right], \quad A_{22} = \frac{F_v}{6E_3 I_3} \cos \beta,$$

$$A_{23} = \left\{ \begin{array}{l} \frac{F_v}{2E_1 I_1} \sin^2 \alpha a + \frac{F_v}{E_1 I_1} \left[ (c+d) \cos \beta + \frac{b}{\sqrt{2}} \right] a + \frac{F_v}{E_2 I_2} \left[ (c+d) b \cos \beta + \frac{b^2}{2\sqrt{2}} \right] + \\ \frac{F_v}{E_3 I_3} c \left[ (c+d) \cos \beta + \frac{c}{2} \right] + \frac{F_v d e}{2G_a I_a} \end{array} \right\} \cos \beta,$$

$$A_{24} = \frac{F_v d' \cos \beta}{2E_a I_a}, \quad A_{25} = \frac{F_v \cos \beta}{12E_a I_a}.$$

$$K_{v_p} = \frac{F_h}{[B_5 + A_{20}c + A_{21}c^2 - A_{22}c^3]} : \text{vertical bending stiffness of the pillar.}$$

$$A_{26} = \sin \alpha, \quad A_{27} = \frac{1}{\sqrt{2}}, \quad A_{28} = a \sin \alpha, \quad A_{29} = \frac{b}{\sqrt{2}} + a \sin \alpha, \quad A_{30} = \cos \beta, \quad A_{31} = A_{29} + c \cos \beta.$$

### III vibration mode - Torsional deformation

$$K_{\tau_{tor}} = \frac{F_v}{J_{\tau_{tor}}} : \text{torsional bending stiffness of the whole structure},$$

$$\gamma_{\tau_{tor}} = B_7 + d'\theta_2 + A_{34}d'^2 + A_{33}d'^3 : \text{torsional displacement of the handles end},$$

$$\gamma_{\tau_1} = t\theta_1 + A_{32}t^2 - A_{33}t^3 \quad t \in [0, e], \quad \gamma_{\tau_2} = B_7 + s\theta_2 - A_{34}s^2 - A_{33}s^3 \quad s \in [0, d'],$$

$$B_7 = e\theta_1 + A_{32}e^2 - A_{33}e^3, \quad \theta_1 = \frac{F_v(e+d'\sin\gamma)}{G_3 I_3} \left( \frac{b}{\sqrt{2}} + c \right), \quad \theta_2 = \frac{F_v de}{2G_a I_a}, \quad A_{32} = \frac{F_v e}{4E_a I_a},$$

$$A_{33} = \frac{F_v}{12E_a I_a}, \quad A_{34} = \frac{F_v d'}{4E_a I_a}.$$