Deducing the stage-discharge relationship for contracted weirs by the outflow theory of Malcherek

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Abstract

The aim of this paper was the deduction of a new stage-discharge relationship for a contracted weir, having a crest length less than the channel width, by using the Malcherek's outflow theory. The average outflow velocity over the rectangular contracted weir was expressed in terms of the head over weir, the momentum correction coefficient and the ratio between the crest length and the channel width. This theoretically deduced stage-discharge formula was calibrated by measurements carried out for values of the ratio between the crest length and the channel width ranging from 0.3125 to 0.9375. In particular, a relationship to estimate the momentum correction coefficient for contracted weirs was deduced. The analysis showed that the proposed stage-discharge relationship is characterised by an excellent performance and allows to measure discharge values characterised by errors which are, for 96.0% of the measured values, less than or equal to ±5%.

Introduction

Weirs are probably the oldest hydraulic structures made by man for flow measurements (Swamee, 1988). The classic experiments of Bazin, using full-width, thin-plate weirs, were carried out in 1886-1887 (Kindsvater and Carter, 1957).

Kindsvater and Carter (1957) developed a comprehensive analysis of the rectangular thin-plate weirs and proposed a stage-discharge solution which takes into account for the influence of fluid properties and physical characteristic of the weirs and the weir channel.

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Sharp crested weirs (Aydin *et al*., 2002, 2006; Bagheri and Heidarpour, 2010; Ferro, 2012; Bautista-Capetillo *et al*., 2013; Bijankhan *et al*., 2014; Gharahjeh *et al*., 2015) are elevated barriers generally located normal the main flow direction, for forcing the fluid to rise above the obstruction to flow through a regular-shaped opening, which are classically used as flow measuring device. The crest shape, approach channel width, obliquity (angle between the weir crest and the direction normal to the flow motion), and vertical inclination (pivot weir) (Borghei *et al*., 2013; Nikou *et al*., 2016) are the key-parameters affecting the flow over a rectangular weir (Govinda Rao and Muralidhar, 1963; Ferro, 2013; Bijankhan and Ferro, 2017). A weir is classified as sharp-crested when its crest length and the flow head are such that the flow separates and does not attach to the weir before the flow leaves the weir (Rao and Shukla, 1971; Johnson, 2000; Ferro, 2013; Bijankhan and Ferro, 2017). A sharp-crested rectangular weir consists of a vertical thin, generally metal, plate that is placed normally to the flow direction. The rectangular weir is named *contracted* when the width of its crest b is less than the width B of the approach channel while is *non-contracted* for b/B=1. When weirs are used for measuring discharge the structure is characterised by a relationship, named *stage-discharge relationship* (Azimi and Rajaratnam, 2009; Bagheri and Heidarpour, 2010; Aydin *et al*., 2011; Ferro, 2012; Bijankhan *et al*., 2014, 2015; Di Stefano *et al*., 2016) between the discharge Q and the water depth h measured from the reference level (horizontal plane localised at an elevation equal to the weir height P) (Figure 1). These structures were studied using classical physics and experimental analysis to understand the characteristics of flow and to determine the discharge coefficient C_d. This coefficient represents the effects not considered in the derivations of the equations used for estimating discharge from flow depth. Such effects include viscosity, surface tension, velocity distribution in the approach channel, and streamline curvature due to weir contraction (Sargison *et al*., 1972; Ranga Raju *et al*., 1977; De Martino *et al*., 1984; Aydin *et al*., 2011). The ratio of the weir width b to the approach channel width B, named contraction ratio, affects the stage-discharge curve of the weirs (Gharahjeh *et al*., 2015) and using the smaller weir width, it would be possible to measure very low discharges. With this aim, Aydin *et al*. (2002, 2006) proposed the design of the *slit weir*, which is a narrow, vertical, rectangular opening with sharp edges able to measure discharge values less than 0.005 m^3/s. According to Aydin *et al*. (2006) *for slit weir, the channel width must be large enough so that the approach velocity can be ignored*.

Aydin *et al*. (2002, 2006) also stated that the upper bound to ignore the channel width effect is defined by a value of the ratio b/B less than or equal to 0.25. In other words, the *contracted* weirs are characterised by contraction ratio values $0.25 < b/B < 1$ while the *slit* weirs correspond to $b/B \leq 0.25$. For a rectangular normal sharp-crested weir, the well-known stage-discharge relationship is generally derived from energy consideration (Herschy, 1999) or applying the dimensional analysis and the self-similarity theory (Ferro, 2012; Bijankhan *et al*., 2014, 2015; Ferro and Aydin, 2018a).

The solution of the efflux problem, derived by Torricelli (Torricelli, 1644; Malcherek, 2016), is based on statement that the outflow velocity V from a vessel is equal to the terminal velocity of a body falling freely from an height equal to the vessel filling level h (Figure 2):

$$V = \sqrt{2\,g\,h} \tag{1}$$

Then Bernoulli (1738) outlined that the measured outflow velocity is smaller than the theoretical one and introduced, to explain this discrepancy, the concepts of *vena contracta* and discharge coefficient which has to be experimentally determined.

Recently, Malcherek (2016) has developed a new insight in the hydraulic theory of the outflow problem applying the momentum balance in a vessel. In particular, for the stationary situation the following expression of the outflow velocity V is deduced:

$$V = \sqrt{\frac{g\,h}{\beta - \frac{A_A}{A}}} \tag{2}$$

in which β = momentum coefficient which can be assumed equal to 1 for an homogeneous velocity distribution, A = vessel cross-section area and A_A = outflow cross-section area (Figure 2). Eq.(2) is different from the original Torricelli formula (Eq.1) because it does not have the factor 2 under the square root. Integrating the outflow velocity, expressed by Eq.(2), over the weir opening, the following stage-discharge relationship for a rectangular sharp-crested weir placed in a rectangular channel having a channel width B is obtained (Ferro and Aydin, 2018b):

$$Q = \frac{2}{3}\,b\,h\sqrt{\frac{g\,h}{\beta - \frac{b}{B}}} \tag{3}$$

in which g = acceleration due to gravity. Eq.(3) was deduced by Ferro and Aydin (2018b) hypothesing that the ratio A_A/A is independent of water depth h and equal to the ratio b/B. In particular, Ferro and Aydin (2018b) calibrated the theoretically deduced stage-discharge formula (Eq.3) using experimental data obtained for value of the ratio b/B ranging from 0.05 and 0.25, *i.e.* in the range of the slit weirs. In particular, a power empirical relationship linking the momentum coefficient β and the ratio b/B was deduced. This last relationship, substituted into Eq.(3), allowed to obtain a stage-discharge equation characterised by errors which are, for 91% of the measured values, less than or equal to ±5%.

Bijankhan and Mazden (2018) applied the Malcherek's outflow theory for deducing the stage-discharge relationship for rectangular, triangular and circular overflow structures. In particular, for the rectangular weir the solution proposed by Bijankhan and Mazden (2018) can be obtained by Eq.(3) setting $b/B = 0$. In other words, the stage-discharge equation suggested by Bijankhan and Mazden (2018) is a particular case of Eq.(3) and is obtained using an outflow average velocity V which neglects the ratio A_A/A. This choice also affects the estimate of the coefficient β which Bijankhan and Mazden (2018) obtained by Eq.(3) with $b/B = 0$. For rectangular weirs characterised by b/B values ranging from 0.31 to 1, these Authors also used the coefficient β as a fitting parameter. The best agreement between the measured (*h, Q*) pairs and the stage-discharge relationship by Bijankhan and Mazden (2018) (Eq.3 with $b/B = 0$) was obtained for a β value which varies with b/B [see *Figure 2* in the paper by Bijankhan and Mazden (2018)] and it is equal to 1 only for the case of non-contracted (b/B = 1) weir. Thereby, the best fitting β value is close to the value 1.036, deduced by velocity profiles measured by Tsung *et al*. (2014) in a rectangular non-contracted weir, only for the case b/B = 1. For the investigated contracted-weirs, the best-fitting β values estimated by Bijankhan and Mazden (2018) are quite different from both 1 and the value 1.25 suggested by Malcherek (2016). The best-fitting β values estimated by Bijankhan and Mazden (2018) were not tested by velocity measurements in non-contracted weirs. The stage-discharge relationship by Bijankhan and Mazden (2018) (Eq.3 with $b/B = 0$) is characterised by absolute errors that in the best case (β = 1.25) are less than 15% [see *Figure 5* in the paper by Bijankhan and Mazden (2018)].

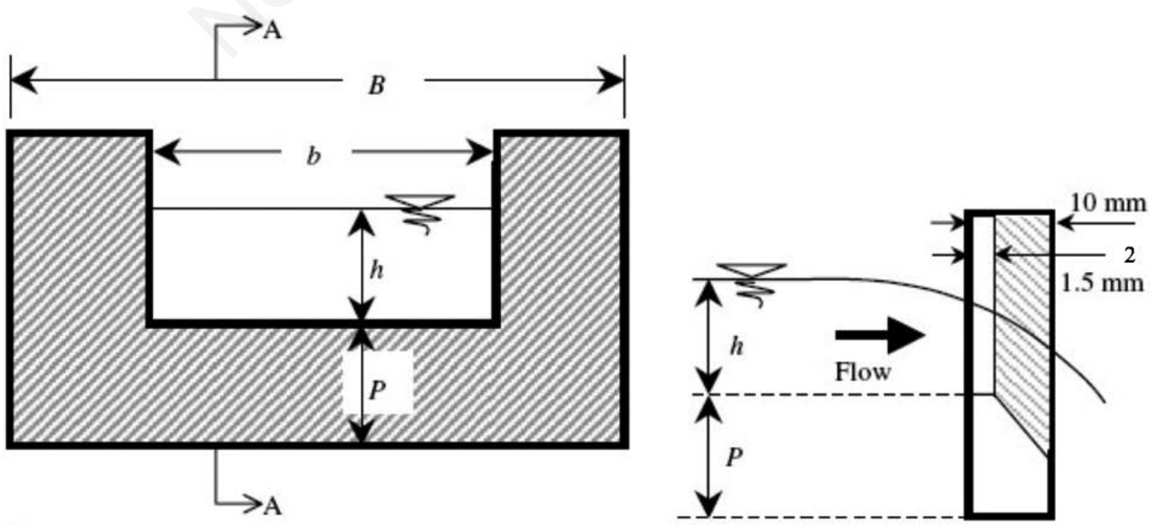

Figure 1. Definition sketch of a contracted weir.

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According to Ferro and Aydin (2018b), rearranging Eq. (3) the following equation is obtained:

$$\frac{Q}{g^{1/2}\, b^{5/2}} = \left(\frac{2}{3}\frac{1}{\sqrt{\beta - \frac{b}{B}}}\right)\left(\frac{h}{b}\right)^{3/2} \tag{4}$$

Setting

$$a = \frac{2}{3}\frac{1}{\sqrt{\beta - \frac{b}{B}}} \tag{5}$$

the stage-discharge relationship assumes the following mathematical shape:

$$Y = aX \tag{6}$$

in which $X = (h/b)^{3/2}$ and $Y = Q/g^{1/2}\, b^{5/2}$.

In this paper, the theoretically deduced stage-discharge relationship (Eq.3) is calibrated using the measurements carried out by Gharahjeh *et al*. (2015) for contracted weirs having values of the ratio *b/B*, ranging from 0.3125 to 0.9375, greater than the upper bound defining the slit weirs. A relationship to estimate the momentum correction coefficient β for contracted weirs which takes into account the ratio *b/B* is also proposed.

Materials and methods

Experimental setup and measuring techniques

The experimental measurements used in this study were carried out by Gharahjeh *et al*. (2015) in the Hydromechanics Laboratory of Civil Engineering Department at Middle East Technical University. A horizontal rectangular channel 6 m long, 0.32 m wide and 0.7 m deep was used for the tests. The weir plate is 10-mm thick and the bevelled edges of the weir plate are 2-mm thick. The approach channel is smooth, made of plexiglass and upstream of it there is tank with screens inside to still the water. Water was supplied from upstream entrance through a pipe. The discharge in the channel was controlled by a valve before it reaches the entrance tank. At the end of the entrance tank there were several vertical parallel screens which subsided the fluctuations generated at the water surface. The measurements were carried out for eleven different weir width (*b*=0.10, 0.12, 0.14, 0.16, 0.18, 0.20, 0.22, 0.24, 0.26, 0.28 and 0.30 m) having the same weir height P=0.1 m. The weir height was chosen taking into account the need, in the experimental runs, to avoid boundary layer influence on the flow over the weir (Bos, 1989). Water level over the weir was measured, at a distance of 2.2 m upstream of the weir section, by a point gauge having an accuracy of 0.1 mm. Discharge was measured by a flow measuring tank located at the end of the channel. Further details on the experimental setup and measurement technique are reported in Gharahjeh *et al*. (2015). Experimental measurements by Gharahjeh *et al*. (2015) used in this investigation are listed in Table 1.

Results and discussion

The coefficent *a* of Eq.(6) is estimated using measurements carried out for eleven different weir width, corresponding to contraction ratio values *b/B* ranging from 0.3125 to 0.9375 (Table 2). Figure 3 shows, as an example for four different values of the contraction ratio (*b/B*=0.3125, 0.5, 0.75 and 0.9375), the agreement between the experimental pairs (*X*, *Y*) and the relationship (Eq.6) between dimensionless discharge and dimensionless head over weir.

Using the measurements carried out in this investigation Eq.(6) was calibrated for each *b/B* value (Table 2). The *a* values listed in Table 2 allow to calculate the β value using the following equation deduced from Eq.(5):

$$\beta = \frac{4}{9\,a^2} + \frac{b}{B} \tag{7}$$

which can be approximated by the following relationship (Figure 4):

$$\beta = 1.3358 + 1.4025\,\frac{b}{B} - 0.7856\left(\frac{b}{B}\right)^2 \tag{8}$$

which can be applied for *b/B* ranging from 0.3125 to 0.9375.

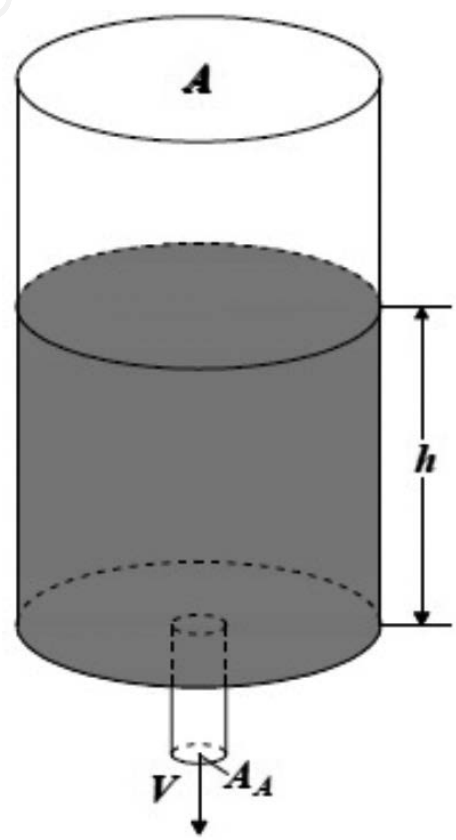

Figure 2. Vertical outflow from a vessel.

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Table 1. Experimental measurements by Gharahjeh *et al.* (2015) used in this investigation.

N	*b* [m]	*h* [m]	*Q* [L s^{-1}]	*N*	*b* [m]	*h* [m]	*Q* [L s^{-1}]	*N*	*b* [m]	*h* [m]	*Q* [L s^{-1}]
1	0.30	0.0109	0.71	45	0.26	0.0976	15.53	89	0.22	0.1315	19.70
2	0.30	0.0308	3.08	46	0.26	0.1071	17.93	90	0.22	0.1466	23.20
3	0.30	0.0421	5.05	47	0.26	0.1201	21.37	91	0.22	0.1606	26.65
4	0.30	0.0501	6.62	48	0.26	0.1321	24.64	92	0.22	0.1741	30.18
5	0.30	0.0576	8.18	49	0.26	0.1411	27.12	93	0.22	0.1889	33.89
6	0.30	0.0646	9.75	50	0.26	0.1506	29.92	94	0.22	0.2001	36.32
7	0.30	0.0723	11.66	51	0.26	0.1616	33.64	95	0.22	0.2131	40.46
8	0.30	0.0804	13.87	52	0.26	0.1706	36.69	96	0.22	0.2216	43.24
9	0.30	0.0916	16.87	53	0.26	0.1786	38.97	97	0.22	0.2321	47.06
10	0.30	0.1011	19.38	54	0.26	0.1856	41.54	98	0.20	0.0186	1.01
11	0.30	0.1131	23.22	55	0.26	0.1954	44.39	99	0.20	0.0334	2.36
12	0.30	0.1231	26.73	56	0.26	0.2036	46.67	100	0.20	0.043	3.36
13	0.30	0.1336	30.62	57	0.24	0.0218	1.52	101	0.20	0.0526	4.49
14	0.30	0.1426	33.39	58	0.24	0.0421	3.84	102	0.20	0.0608	5.52
15	0.30	0.1521	36.91	59	0.24	0.0506	4.96	103	0.20	0.0668	6.16
16	0.30	0.1596	39.85	60	0.24	0.0574	6.20	104	0.20	0.0781	8.10
17	0.30	0.1686	42.96	61	0.24	0.0641	7.30	105	0.20	0.0876	9.58
18	0.30	0.1761	45.29	62	0.24	0.0714	8.68	106	0.20	0.0981	11.34
19	0.30	0.1816	47.39	63	0.24	0.0808	10.46	107	0.20	0.1118	13.79
20	0.28	0.0196	1.53	64	0.24	0.0903	12.49	108	0.20	0.1251	16.41
21	0.28	0.0358	3.72	65	0.24	0.0991	14.41	109	0.20	0.1351	18.41
22	0.28	0.0484	5.77	66	0.24	0.1126	17.28	110	0.20	0.1481	21.10
23	0.28	0.059	7.75	67	0.24	0.1215	19.48	111	0.20	0.1611	23.70
24	0.28	0.0691	9.91	68	0.24	0.1311	21.97	112	0.20	0.1721	26.23
25	0.28	0.0806	12.75	69	0.24	0.1451	25.44	113	0.20	0.1856	29.33
26	0.28	0.0958	16.59	70	0.24	0.1539	28.19	114	0.20	0.1966	31.84
27	0.28	0.1073	19.41	71	0.24	0.1636	30.64	115	0.20	0.2106	35.80
28	0.28	0.1231	23.91	72	0.24	0.1746	33.74	116	0.20	0.2261	39.62
29	0.28	0.1341	27.12	73	0.24	0.1861	37.40	117	0.20	0.2396	43.32
30	0.28	0.1456	31.23	74	0.24	0.1976	40.90	118	0.20	0.2476	45.69
31	0.28	0.1551	34.88	75	0.24	0.2068	42.97	119	0.18	0.0204	1.04
32	0.28	0.1636	37.63	76	0.24	0.2156	45.34	120	0.18	0.0325	2.01
33	0.28	0.1696	39.24	77	0.22	0.0201	1.22	121	0.18	0.0451	3.20
34	0.28	0.1761	41.17	78	0.22	0.0269	1.85	122	0.18	0.0559	4.41
35	0.28	0.1826	43.31	79	0.22	0.0334	2.56	123	0.18	0.0636	5.28
36	0.28	0.1886	45.33	80	0.22	0.0411	3.47	124	0.18	0.0711	6.25
37	0.28	0.1961	47.02	81	0.22	0.0433	3.76	125	0.18	0.0785	7.23
38	0.26	0.0206	1.62	82	0.22	0.0536	5.10	126	0.18	0.0894	8.81
39	0.26	0.0376	3.64	83	0.22	0.0606	5.93	127	0.18	0.1005	10.53
40	0.26	0.0481	5.24	84	0.22	0.0714	7.82	128	0.18	0.1171	13.28
41	0.26	0.0551	6.48	85	0.22	0.0814	9.57	129	0.18	0.1286	15.24
42	0.26	0.0631	7.94	86	0.22	0.0931	11.79	130	0.18	0.1421	17.78
43	0.26	0.0731	9.86	87	0.22	0.1051	14.10	131	0.18	0.1571	20.79
44	0.26	0.0836	12.08	88	0.22	0.1201	17.31	132	0.18	0.1718	23.64

Continued on next page.

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Table 1. Continued from previous page

N	*b* [m]	*h* [m]	*Q* [L s^{-1}]	*N*	*b* [m]	*h* [m]	*Q* [L s^{-1}]	*N*	*b* [m]	*h* [m]	*Q* [L s^{-1}]
133	0.18	0.1866	26.81	177	0.14	0.2359	28.30	221	0.10	0.1024	5.93
134	0.18	0.1976	28.78	178	0.14	0.2572	32.81	222	0.10	0.0908	4.93
135	0.18	0.2153	32.72	179	0.14	0.2712	35.53	223	0.10	0.079	4.06
136	0.18	0.2296	36.64	180	0.14	0.2887	38.92	224	0.10	0.0614	2.83
137	0.18	0.2438	39.83	181	0.14	0.3017	41.08	225	0.10	0.0444	1.74
138	0.18	0.2596	42.67	182	0.14	0.3232	45.89	226	0.10	0.0217	0.64
139	0.18	0.2676	43.91	183	0.12	0.0192	0.65				
140	0.16	0.0217	0.94	185	0.12	0.0447	2.05				
141	0.16	0.027	1.32	185	0.12	0.0672	3.77				
142	0.16	0.0436	2.67	186	0.12	0.076	4.56				
143	0.16	0.0582	4.00	187	0.12	0.0876	5.58				
144	0.16	0.0687	5.10	188	0.12	0.0987	6.87				
145	0.16	0.0807	6.40	189	0.12	0.1095	7.79				
146	0.16	0.0939	8.07	190	0.12	0.1236	9.32				
147	0.16	0.1122	10.55	191	0.12	0.144	11.75				
148	0.16	0.1309	13.33	192	0.12	0.1646	14.42				
149	0.16	0.1459	15.78	193	0.12	0.1825	16.94				
150	0.16	0.1576	17.74	194	0.12	0.1977	18.83				
151	0.16	0.1732	20.43	195	0.12	0.2137	21.43				
152	0.16	0.1877	23.13	196	0.12	0.2305	23.67				
153	0.16	0.2017	25.87	197	0.12	0.2487	26.45				
154	0.16	0.2202	29.76	198	0.12	0.2662	29.55				
155	0.16	0.2312	32.00	199	0.12	0.2808	31.57				
156	0.16	0.2432	34.50	200	0.12	0.2992	34.72				
157	0.16	0.2507	36.21	201	0.12	0.3147	38.32				
158	0.16	0.2627	38.87	202	0.12	0.3287	40.00				
159	0.16	0.2747	41.46	203	0.12	0.3482	44.11				
160	0.16	0.2857	43.58	204	0.10	0.4167	47.45				
161	0.16	0.2957	45.65	205	0.10	0.4046	45.56				
162	0.14	0.0157	0.56	206	0.10	0.3812	41.56				
163	0.14	0.0402	2.10	207	0.10	0.3607	38.88				
164	0.14	0.0542	3.15	208	0.10	0.3387	34.89				
165	0.14	0.0652	4.36	209	0.10	0.3177	31.73				
166	0.14	0.0733	4.85	210	0.10	0.2957	28.71				
167	0.14	0.0852	6.14	211	0.10	0.2722	25.49				
168	0.14	0.0989	7.61	212	0.10	0.2487	22.33				
169	0.14	0.1122	9.25	213	0.10	0.2287	19.98				
170	0.14	0.1257	11.02	214	0.10	0.2127	17.96				
171	0.14	0.1398	12.90	215	0.10	0.1955	15.87				
172	0.14	0.1547	15.05	216	0.10	0.1798	13.89				
173	0.14	0.1732	17.89	217	0.10	0.1637	12.08				
174	0.14	0.1875	20.33	218	0.10	0.1493	10.49				
175	0.14	0.2047	23.07	219	0.10	0.1327	8.80				
176	0.14	0.2177	25.14	220	0.10	0.1177	7.40				

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The discharge values calculated by Eq.(3), with β estimated by Eq.(8), and the measured ones are compared in Figure 5.

The deduced stage-discharge relationships allow to calculate discharge values Q_c which are characterised by errors E:

$$E = 100\left(\frac{Q_c - Q_m}{Q_m}\right) \tag{9}$$

which are, for 96.0% of the measured discharge values Q_m, less than or equal to ±5%. As depicted in Figure 6, 76.1% of the errors is less than or equal to ±2%.

In conclusion Eq.(3) coupled with Eq.(8) is characterised by errors which are less than those obtained by applying the stage-discharge relationship of Bijankhan and Mazden (2018).

Conclusions

A rectangular weir is named *contracted* when the width of its crest b is less than the width B of the approach channel. In this paper the stage-discharge relationship of a contracted weir was theoretically deduced, by using the outflow theory of Malcherek, taking into account the effect of the ratio b/B on the average outflow velocity.

The proposed stage-discharge equation (Eq.3) takes into account the head over weir, the momentum correction coefficient β and the ratio between the crest length b and the channel width B. For estimating the β coefficient some flume measurements, carried out for values of the ratio b/B ranging from 0.3125 to 0.9375, were used. The measured β coefficient values (Eq.7) were related to the ratio b/B using a quadratic function (Eq.8). The proposed stage-discharge relationship (Eq.3), in which β coefficient is estimated by the Eq.(8), allows to calculate discharge values characterised by errors which are, for 96.0% of the measured values, less than or equal to ±5%. Furthermore, Eq.(3) coupled with Eq.(8) is characterised by errors which are less than those obtained by applying the stage-discharge relationship of Bijankhan and Mazden (2018).

Results indicate that the average velocity over a contracted rectangular weir can be given in terms of head over weir h and the contraction ratio b/B.

Table 2. Values of *a* and β coefficients.

b [m]	*b/B*	*a*	β
0.10	0.3125	0.5687	1.687
0.12	0.3750	0.5700	1.743
0.14	0.4375	0.5688	1.811
0.16	0.5000	0.5713	1.862
0.18	0.5625	0.5800	1.884
0.20	0.6250	0.5889	1.907
0.22	0.6875	0.6007	1.919
0.24	0.7500	0.6126	1.934
0.26	0.8125	0.6312	1.928
0.28	0.8750	0.6342	1.980
0.30	0.9375	0.6564	1.969

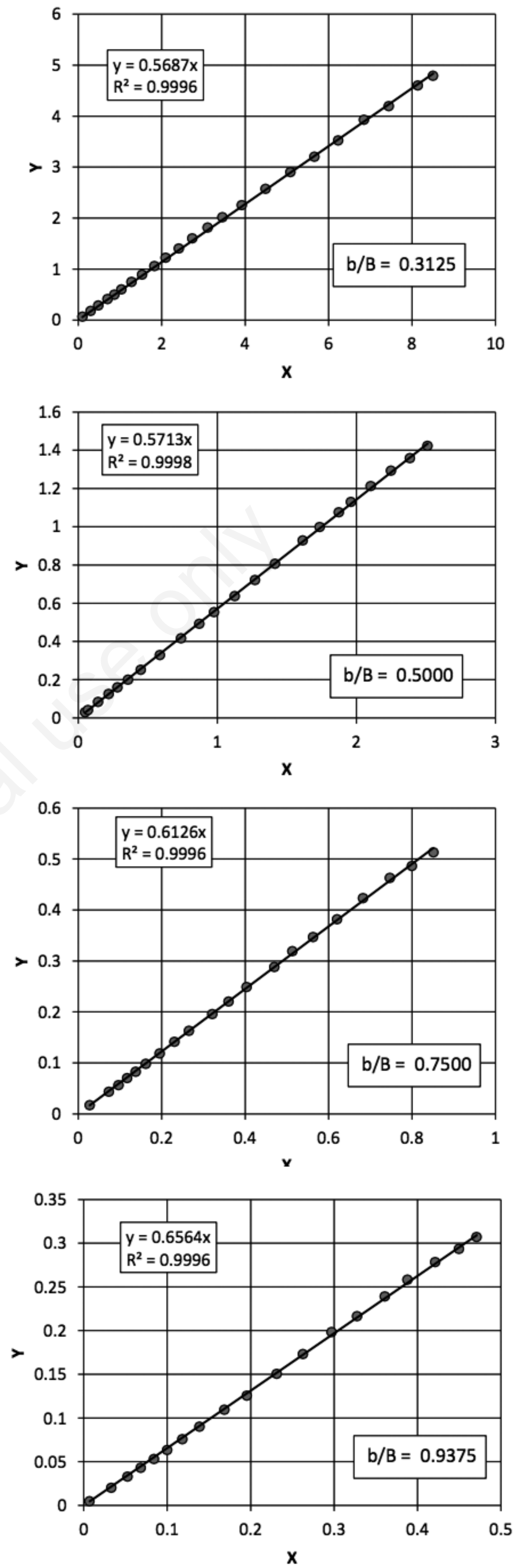

Figure 3. Relationship between dimensionless discharge and dimensionless head over weir for four values of the contraction ratio (*b/B* = 0.3125, 0.5, 0.75 and 0.9375).

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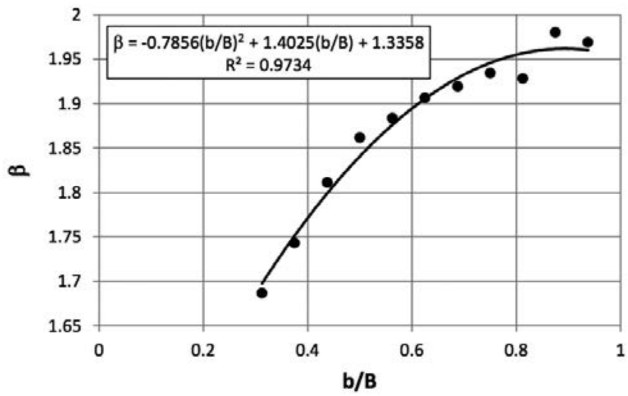

Figure 4. Coefficient β as function of contraction ratio.

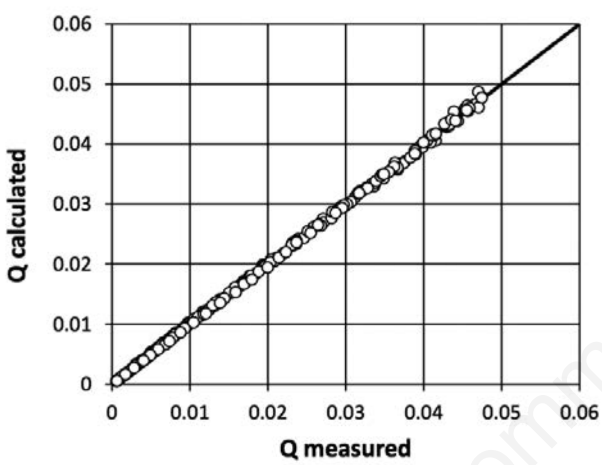

Figure 5. Comparison between measured discharges and the ones calculated by Eq.(3) with β estimated by Eq. (8).

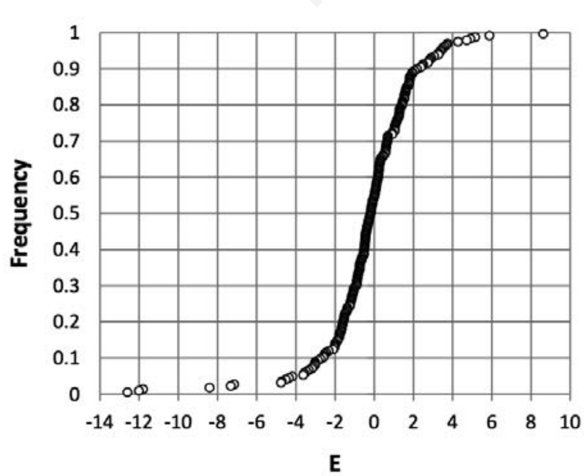

Figure 6. Frequency distribution of the errors E.

References

Aydin I., Altan-Sakarya A.B., Ger A.M. 2006. Performance of slit weir. J. Hydraul. Eng. ASCE 132:987-9.

Aydin I., Altan-Sakarya A.B., Sisman C. 2011. Discharge formula for rectangular sharp-crested weirs. Flow Meas. Instrum. 22:144-51.

Aydin I., Ger A.M., Hincal O. 2002. Measurement of small discharges in open channels by slit weir. J. Hydraul. Eng. ASCE 128:234-7.

Azimi A., Rajaratnam N. 2009. Discharge characteristics of weirs of finite crest length. J. Hydr. Engine. ASCE 135:1081-5.

Bagheri S., Heidarpour M. 2010. Flow over rectangular sharp-crested weirs. J. Irr. Sci. 28:173-9.

Bautista-Capetillo C., Roble O., Júnez-Ferreira H., Playán E. 2013. Discharge coefficient analysis for triangular sharp-crested weirs using low-speed photographic technique. J. Irrig. Drain Eng. ASCE 139:1-4.

Bernoulli D. 1738. Hydrodynamica sive de viribus motibus fluidorum commentarii. Johann Reinhold Dulsecker, Strassburg, Austria.

Bijankhan M., Di Stefano C., Ferro V., Kouchakzadeh S. 2014. New stage-discharge relationship for weirs of finite crest length. J. Irrig. Drain Eng. ASCE 140:1-8.

Bijankhan M., Di Stefano C., Ferro V., Kouchakzadh S. 2015. Closure to "New stage-discharge relationship for weirs of finite crest length". J. Irrig. Drain Eng. ASCE 141:1-3.

Bijankhan M., Ferro V. 2017. Dimensional analysis and stage-discharge relationship for weirs: a review. J. Agricult. Engine. XLVIII:1-11.

Bijankhan M., Mazdeh A.M. 2018. Assessing Malcherek's outflow theory to deduce the theoretical stage-discharge formula for overflow structures. J. Irrig. Drain Eng. ASCE 144:1-11.

Borghei S.M., Vatannia Z., Ghodsian M., Jalili M.R. 2013. Oblique rectangular sharp-crested weir. Proc. ICE Water & Maritime Engineering 156:185-91.

Bos M.G. 1989. Discharge measurement structures. International Institute for Land Reclamation and Improvement (ILR), Wageningen, The Netherlands.

De Martino G., Ragone A. 1984. Effects of viscosity and surface tension on slot weirs flow. J. Hydr. Res. 22:327-41.

Di Stefano C., Ferro V., Bijankhan M. 2016. New theoretical solution of the outflow process for a weir with complex shape. J. Irrig. Drain Eng. ASCE 142:1-9.

Ferro V. 2012. New theoretical solution of the stage-discharge relationship for sharp-crested and broad weirs. J. Irrig. Drain Eng. ASCE 126:190-3.

Ferro V. 2013. Closure to "New theoretical solution of the stage-discharge relationship for sharp-crested and broad weirs by V. Ferro". J. Irrig. Drain Eng. ASCE 139:518-20.

Ferro V., Aydin I. 2018a. New theoretical solution of the stage-discharge relationship for slit weirs. J. Irrig. Drain Eng. ASCE, 144 3, 06018001-1-7.

Ferro V., Aydin I. 2018b. Testing the otflow theory of Malcherek by slit weir data. Flow Meas. Instrum. 59:114-7.

Gharahjeh S., Aydin I., Altan-Sakarya A.B. 2015. Weir velocity formulation for sharp-crested rectangular weirs. Flow Meas. Instrum. 41:50-6.

Govinda Rao N.S., Muralidhar D. 1963. Discharge characteristics of weirs of finite-crest width. La Houille Blanche 5:537-45.

Herschy H. 1999. Hydrometry. Principles and practices. Wiley, New York, NY, USA, pp 55-60.

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Johnson M.C. 2000. Discharge coefficient analysis for flat-topped and sharp-crested weirs. Irrig. Sci. 19:133-7.

Kindsvater C.E., Carter R.W. 1957. Discharge characteristics of rectangular thin-plate weirs. J. Hydraul. Div. ASCE 145:1-36.

Malcherek A. 2016. History of the Torricelli principle and a new outflow theory. J. Hydraul. Eng. ASCE 142:1-7.

Nikou N.S.R., Monem M.J., Safavi K. 2016. Extraction of the flow rate equation under free and submeged flow conditions in pivot weirs with different side contractions. J. Irrig. Drain Eng., ASCE, 142, 04016025, 1-8.

Ranga Raju K.G., Asawa G.L. 1977. Viscosity and surface tension effects on weir flow. J. Hydr. Div. ASCE 103:1227-31.

Rao S.S., Shukla M.J. 1971. Characteristics of flow over weirs of finite crest width. J. Hydr. Div. ASCE 97:1807-16.

Sargison EJ. 1972. The influence of surface tension on weir flow. J. Hydr. Res. 10:431-46.

Swamee P.K. 1988. Generalised rectangular weir equations. J. Hydraul. Eng. ASCE 114:945-9.

Torricelli E. 1644. Opera geometrica. Amatoris Masse & Laurentij de Landis, Florence, Italy.

Tsung S., Lai J., Young D.L. 2014. Velocity distribution and discharge calculation at a sharp-crested weir. Puddy Water Environ. 12:203-12.

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